AN AGENT-BASED MODEL FOR PORTFOLIO OPTIMIZATIONS IN A TIGHT PROBLEM

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Portfolio optimization is to optimize the weights of assets included in a portfolio in order to achieve the investment objective. This optimization problem is one of the combinatorial optimization problems, and it is hard to solve in reasonable time as the number of assets or the number of weights of each asset exceeds not so very large numbers. On the other hand, it is well known that Information Ratio (IR) appropriately measures the performance of active managed portfolios. Even we employ IR as an objective function; however, the portfolio optimization can be viewed as a tight problem that has a lot of solutions near the optimal solution in a narrow solution space. It is hard to solve such a tight problem even by mighty evolutional algorithms. In order to solve such a problem, we propose an Agent-based Model in this paper. Our agent has one portfolio, IR and its character as a set of properties. In the population, there is one leader agent and the others are follower agents. Through the progress of processing the population, the agent properties change by the interaction between the leader and the follower, and then we obtain the leader who has the highest IR. The numerical experiments show that our model works well for the portfolio optimization problem though the effectiveness of the characters of our agents depends on the data period.

1. Introduction

Portfolio optimization is to optimize the weights of assets included in a portfolio in order to achieve the investment objective. This optimization problem is one of the combinatorial optimization problems, and it is hard to solve in reasonable

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time as the number of assets or the number of weights of each asset exceeds not so very large numbers. Many researchers have tackled this problem by using methods based on evolutional algorithms (for this, see e.g., [1,2]). It is well known that Information Ratio (IR), which is built on the modern portfolio theory proposed by Markowitz [3], appropriately measures the performance of active managed portfolios. Even we employ IR as an objective function; however, the portfolio optimization can be viewed as a "*tight problem*" that has a lot of solutions near the optimal solution in a narrow solution space. Orito et al. [4] reported that the portfolio optimization problem is one of the tight problems and it is hard to solve even by the mighty evolutional algorithms.

On the other hand, agent-based models for artificial financial markets are recently popular in the research field. The agent properties change by the interaction between agents, and gradually a best performing agent is emerging as an optimal solution (e.g. see [5,6]). Based on the idea of agent-based models, we propose an Agent-based Model for the portfolio optimizations in this paper. Generally, many agent-based models describe interactions and dynamics in a group of traders on an artificial financial market [7]. Our Agent-based Model is, however, adopted as a global and a local search methods for the portfolio optimizations. Our agent has a set of properties. They are one portfolio, the IR obtained by the portfolio and its character. In the population, there is one leader agent, and the others are follower agents. Some followers obediently follow the leader's behaviors, but the others are not so obedient. Some of them are disobedient and taking the opposite behaviors against the leader, and the others determine their behaviors quite independently. Through the progress of processing the population, the agent properties change through the interaction between the leader and the followers, and gradually a best performing agent (the leader agent) with the highest IR is emerging as the optimal solution.

2. Portfolio Optimization Problem

First, we define the following notations for the portfolio optimization problem.

N : total number of all the assets included in a portfolio.
i : Asset i , $i = 1, \cdots, N$.
M : total number of all the units of investment.
M_i : the unit of investment for Asset i . That is an integer such that $\sum M_i = M$.
g_i : the weight of Asset i . That is a real number $g_i = M_i / M \ (0 \le g_i \le 1)$.

$\mathbf{G}_k$: the portfolio of k -th agent in a population. That is the vector $\mathbf{G}_k = (g_1, \cdots, g_N)$.

$\mathbf{P}_{index}$: the sequence of the rates of changes of benchmark index over $t = 1, \cdots, T$. That is the vector $\mathbf{P}_{index} = (P_{index}(1), \cdots, P_{index}(T))$.

$\mathbf{P}_{\mathbf{G}_j}$: the sequence of the return rates of portfolio $\mathbf{G}_k$ over $t = 1, \cdots, T$. That is the vector $\mathbf{P}_{\mathbf{G}_k} = (P_{\mathbf{G}_k}(1), \cdots, P_{\mathbf{G}_k}(T))$.

$IR_{\mathbf{G}_k}$: IR obtained by the portfolio $\mathbf{G}_k$. This is defined by Eq. (1) as our objective function.

$$\max \quad IR_{\mathbf{G}_k} = \frac{E[\mathbf{P}_{\mathbf{G}_k} - \mathbf{P}_{index}]}{\sqrt{\mathrm{var}[\mathbf{P}_{\mathbf{G}_k} - \mathbf{P}_{index}]}}, \tag{1}$$

where $E[\mathbf{P}_{\mathbf{G}_k} - \mathbf{P}_{index}]$ is the expected value of the historical data of portfolio's return rates over or under the rates of changes of benchmark index, and $\sqrt{\mathrm{var}[\mathbf{P}_{\mathbf{G}_k} - \mathbf{P}_{index}]}$ is the standard deviation from the same data.

In this paper, we optimize the combination of weights of N assets with M elements included in the portfolio in order to maximize IR. The number of combinations can be calculated as $(M + N - 1)!/N!(M - 1)!$.

3. Agent-based Model

Fig. 1 shows a typical agent. Each agent has one portfolio, IR, and its character as the properties. The character of agent is either "*obedient*", "*disobedient*", or "*independent*."

Agent k	Portfolio: $\mathbf{G}_k = (g_1, \cdots, g_N)$
	Unit of investment of Porttfolio: $(M_1, \cdots, M_N)$
	Information Ratio: $IR_{\mathbf{G}_k}$
	Character: Obedient/Disobedient/Independent

Figure 1. Agent properties.

Let s be the number of progresses of processing of the population. On the first ($s = 1$) progress, we set W agents as the initial population. For each unit of investment of Agent k , an integer M_i is randomly given. So Agent k has its portfolio $\mathbf{G}_k$ and its IR. In addition, Agent k is randomly given one of the following characters: obedient, disobedient or independent. The rates of the numbers of obedient agents, disobedient agents and independent agents in the population are α , β and γ , respectively. On the s -th progress, the agent properties change through the interaction between the leader and the follower agents. The agent with the highest IR becomes the leader and the others remain to be followers in the population. The behaviors of the leader and each kind of the followers are as follows.

- **Leader**

Leader is an agent whose IR is the highest of all the agents. The portfolio of the leader is represented as $\mathbf{G}_k * = (g_1 *, \cdots, g_N *)$. Let $M_{i_j} *^{(s)}$ be the unit of investment of the Asset i with the j -th largest unit on the s -th progress. The weight of the Asset i depends on $M_{i_j} *^{(s)}$. Thus it is represented as $g_{i_j} *^{(s)}$. In order to update the properties of the agents through the interactions between the leader and the followers, we choose n highest assets in the order of weights from the leader's portfolio as $M_{i_1} *^{(s)} \geq M_{i_2} *^{(s)} \geq \cdots \geq M_{i_n} *^{(s)} \geq \cdots \geq M_{i_N} *^{(s)}$.

- **Obedient follower**

Obedient followers are agents that imitate the leader's properties. When the unit of investment M_{i_j} $(j \leq n)$ of an obedient follower is smaller than that of the leader, the number of the follower's unit is increased, and if the unit of investment of the follower is larger than that of the leader, it is decreased in order to make the follower's number close to the leader's. This operation is defined by Eq. (2) as follows:

$$M_{i_j}^{(s+1)} = \begin{cases} M_{i_j}^{(s)} - X & \text{if } M_{i_j}^{(s)} > M_{i_j} *^{(s)} \\ M_{i_j}^{(s)} & \text{if } M_{i_j}^{(s)} = M_{i_j} *^{(s)} \\ M_{i_j}^{(s)} + X & \text{if } M_{i_j}^{(s)} < M_{i_j} *^{(s)} \end{cases} \quad (j = 1, \cdots, n), \tag{2}$$

where X is a given integer parameter. The obedient followers' behavior is corresponding to a local search.

- **Disobedient follower**

Disobedient followers are agents that take the opposite behaviors against the leader. When the unit of investment M_{i_j} $(j \leq n)$ of a disobedient follower is smaller than that of leader, the number of unit is decreased, and if the unit of investment of the follower is larger than that of the leader, it is increased in order to make the follower's number distant from the leader's. If the number of unit of a follower is the same as the leader's, increasing or decreasing number of unit is randomly determined. This operation is defined by Eq. (3) as follows:

$$M_{i_j}^{(s+1)} = \begin{cases} M_{i_j}^{(s)} + X & \text{if } M_{i_j}^{(s)} > M_{i_j}^{*(s)} \\ M_{i_j}^{(s)} + X \text{ or } M_{i_j}^{(s)} - X & \text{if } M_{i_j}^{(s)} = M_{i_j}^{*(s)} \\ M_{i_j}^{(s)} - X & \text{if } M_{i_j}^{(s)} < M_{i_j}^{*(s)} \end{cases} \quad (j = 1, \cdots, n), \tag{3}$$

where X is a given integer parameter. The disobedient followers' behavior is corresponding to a global search.

- **Independent follower**

Independent followers are agents whose behaviors are not influenced by the leader, and change their properties by their own independent rule defined as Eq. (4). A randomly chosen integer $M_{i_j}^{(s+1)}$ is set for the unit of investment.

$$M_{i_j}^{(s+1)} \in [0, V] \quad (j = 1, \cdots, n), \tag{4}$$

where V is a given integer parameter. The independent followers' behavior is corresponding to a random search.

After changing the properties of the followers, a normalization process randomly selects assets and increases or decreases the units until the current number of units of each of followers is equal to the given total number of all the units M. Finally, the new weight of asset $g_i = M_i / M$ is calculated again and IR of the portfolio is updated in order to complete the updates of all the properties of each follower.

Through the progress of processing the population, some followers may have higher IR than the leader. Then, we take the follower with the highest IR and make it the new leader. After certain iterations, and the terminate condition met, we should have the optimal solution as the leader that has the highest IR.

4. Numerical Experiments

We have applied our Agent-based Model to each of 12 data periods on the 1st Section of Tokyo Stock Exchange from Jan. 6, 1997 to Oct. 2, 2006. Each data period is 100 days, and is shifted every 200 days from Jan. 6, 1997. The dataset is a subset of TOPIX which is a well known benchmark index in the market. The parameters used in our model are shown as follows; Total number of assets: N = 100, Total number of all the units of investment: M = 10000, Number of agents: W = 100, Number of the iterations for the processes = 1000, Number of assets for the changes in Eqs. (2), (3) and (4): n = 20, Integer parameter for Eqs.

(2) and (3): X = 5, Integer parameter for Eq. (4): V = 20, Obedient agents rate: α = 0.8, Disobedient agents rate: β = 0.1 and Independent agents rate: γ = 0.1.

For each period, IRs of the leader on the 10th, 100th, 1000th and 10000th progresses are shown in Table 1. In order to demonstrate the efficiency of our model, we compare the results with the results produced by Random Model, which is a model that uses only random setting for all agents. We note that IR of the leader on the m-th progress in our model is comparable with the best IR of $m \times W$ agents for Random Model. Thus, the best IRs of 1000, 10000, 100000 and 1000000 agents for Random Model are also shown in Table 1.

Table 1. IR.

Data	Agent-based Model				Random Model			
Period	10th	100th	1000th	10000th	1000	10000	100000	1000000
1	0.2357	0.2443	0.3098	0.3979	0.2359	0.2359	0.2375	0.2388
2	0.0212	0.0465	0.1081	0.1864	0.0169	0.0216	0.0221	0.0235
3	0.2302	0.2646	0.3856	0.4444	0.2305	0.2247	0.2270	0.2295
4	0.2772	0.3234	0.3829	0.4504	0.2743	0.2778	0.2815	0.2831
5	0.2522	0.3181	0.4051	0.5020	0.2437	0.2588	0.2538	0.2586
6	0.0782	0.1595	0.2531	0.3922	0.0773	0.0950	0.0986	0.0976
7	0.0872	0.1183	0.2934	0.3796	0.0880	0.0905	0.0945	0.0964
8	0.0618	0.0803	0.2094	0.3455	0.0629	0.0673	0.0687	0.0723
9	0.1766	0.2024	0.2615	0.3671	0.1739	0.1753	0.1805	0.1822
10	-0.0704	-0.0295	0.0696	0.1449	-0.0708	-0.0707	-0.0673	-0.0650
11	-0.0094	0.0347	0.0861	0.1535	-0.0177	-0.0151	-0.0088	-0.0087
12	-0.0460	-0.0180	0.0375	0.1670	-0.0478	-0.0442	-0.0417	-0.0385

From Table 1, IRs of our model after the 100th progress are higher than those of Random Model in the entire data periods. This means that our model works well to find the better portfolios compared with the random search. The value of IR obtained by our model, however, depends on the data period. For discussion on the problem, the characters of the leaders on the 10th, 100th, 1000th and 10000th progresses are shown in Table 2.

We can observe from Table 2 that the leader on the 10000th progress is obedient agent for the Period 1, 3, 4, 5, 6, 7, 8 and 9 and is disobedient agent for the Period 2, 10, 11 and 12, respectively. In addition, IRs of all the disobedient agents are lower than those of all the obedient agents from Table 1. As mentioned in Section 3, an obedient agent's search and a disobedient agent's search are corresponding to the local search and the global search, respectively. This means that the obedient agents become effective agents as a local search in the group of high IR's agents. On the other hand, the disobedient agents become

Table 2. The character of the leader.

Data Period	10th	100th	1000th	10000th
1	Obedient	Obedient	Obedient	Obedient
2	Obedient	Obedient	Obedient	Disobedient
3	Obedient	Obedient	Obedient	Obedient
4	Obedient	Obedient	Obedient	Obedient
5	Obedient	Obedient	Obedient	Obedient
6	Obedient	Obedient	Obedient	Obedient
7	Obedient	Obedient	Obedient	Obedient
8	Obedient	Obedient	Obedient	Obedient
9	Obedient	Obedient	Obedient	Obedient
10	Obedient	Obedient	Disobedient	Disobedient
11	Obedient	Obedient	Disobedient	Disobedient
12	Obedient	Obedient	Disobedient	Disobedient

effective agents as a global search in the group of low IR's agents.

5. Conclusions

In this paper, we proposed an Agent-based Model for the portfolio optimization problem. The numerical experiments demonstrated that our model works well to find the better portfolio than randomly selected one. To investigate the detail relations between our model and a local/global search is our future direction.

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