Leveraging the Flight to Quality: Maximizing Diversification of High-Risk High-Return Stocks Over Short Time Periods With Limited-Cardinality Portfolios

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Over the long term, more volatile investments tend to have higher returns. This paper explores that relationship over shorter timescales. A novel approach to portfolio optimization seeks to maximize diversification, and exploit the volatility-return relationship. As an investment strategy this works, in that it returns above-index returns over the test period. However, the relationship over short timescales is much weaker than over longer timescales.

Keywords: Value at risk; VaR; Portfolio optimization; Limited cardinality.

1. Motivation: Greater Returns = Greater Risk, Sometimes

The cliché is that the greater the risk, the greater the return. You can either sleep well or eat well but not both. We show our students and our financial planning customers graphs like Figure 1, and emphasize diversification; you can mitigate the volatility of those high-yielding stocks, so long as the ups and downs cancel out somewhat.

Figure 1 comes with fine print. Graphs like this usually average risk and return over a whole year, and entire classes of investments.

What about shorter time periods? What about individual stocks, rather than whole categories of investments? For 2 weeks in February 2008, Figure 2 shows the risk-return of the member companies of the S&P 500.

In Figure 2, the usual risk-return relationship is reversed, and greater volatility gave lower returns. Admittedly this relationship is rather weak, with an adjusted R-squared of 0.1835, but the usual risk-return relationship is reversed: more volatile stocks gave lower returns.

What could cause this reversal in Figure 2? One possibility is a "flight to quality". The market was paying a premium for big, stable stocks and

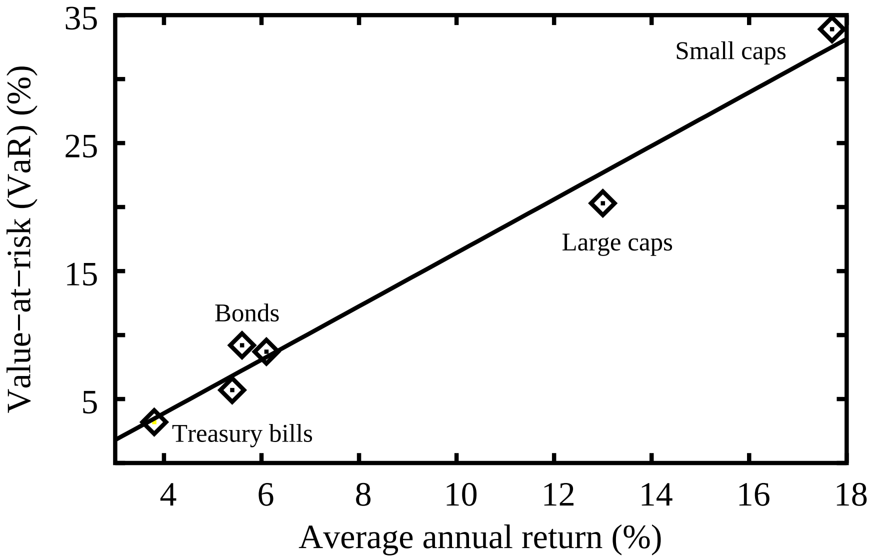

Fig. 1. Adapted from Malkiel,[1] showing the relationship between volatility and return over the long term, in this case the whole twentieth century for entire investment classes.

avoiding smaller, riskier companies that would normally be more attractive.

In Figure 2 and for the rest of this paper, we calculate Value-at-Risk (VaR) by Monte Carlo simulation of 50,000 trading days using a covariance matrix found from Cholesky decomposition of the most recent 130 historical trading days (about six months). Return is how far the stock (or portfolio of stocks) goes up or down in the two-week period, from closing price to closing price. To avoid survivor bias, we take the S&P 500 index as it was on the starting date of each two-week period, and take into account dividends, stock splits, delistings, and mergers/acquisitions during each period.

Let us look at a sample of 2-week periods in 2004-2007, and find the Spearman's rank correlation coefficient for the relationship between return and Value-at-Risk. For 34 different 2-week periods, about one-third had no significant correlation. The remaining two-thirds of these periods had correlation different from zero (with 95% confidence). These are shown in Figure 3. Those correlations, while non-zero, are not particularly strong.

Figure 3 shows that when the S&P 500 is rising, the correlation between risk and return (though weak) is usually positive, agreeing with conventional wisdom and with Figure 1. However, there are still quite a few periods in Figure 3 where the relationship is reversed, and correlation is negative (though still weak). That is, risky stocks can give lower returns

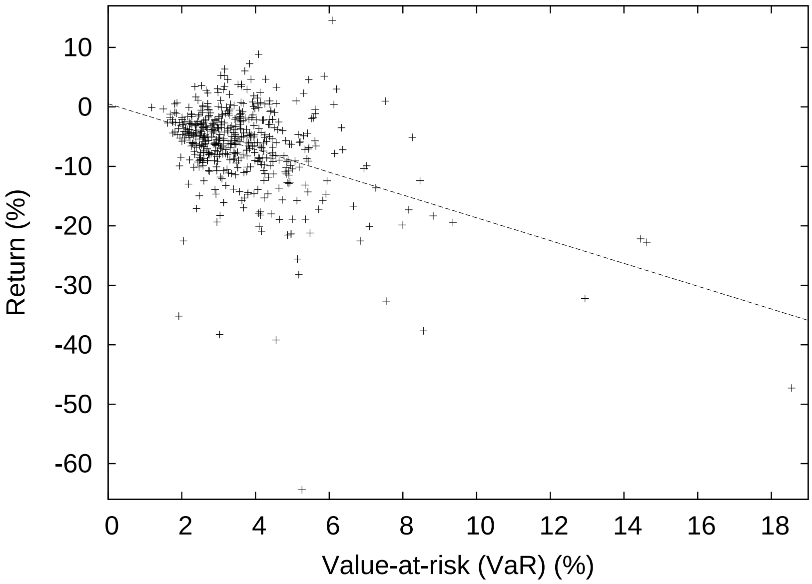

Fig. 2. For the two weeks starting on 28 February 2008, this shows the relationship between volatility and return for the member stocks of the S&P 500. Here, higher volatility gave *lower* returns, not higher. Correlation is negative, but weak (Spearman ρ = -0.272).

than stable stocks, especially when market is declining. This agrees with the notion of a "flight to quality" in a falling market, causing the risk-return relationship to reverse. Linear regression on Figure 3 suggests a relationship exists, at the 99% confidence level.

The risk-return relationship over 2-week periods can vary from positive to negative. So perhaps a feasible trading strategy would be to identify a portfolio with maximal diversification, where the portfolio as a whole was much less volatile than its members.

Optimization of portfolios with limited cardinality (and other realistic assumptions) is NP-complete.[2] Limited cardinality makes for a more challenging optimization problem.

2. A Novel Step Beyond Markowitz

This paper seeks to find portfolios which maximize diversification, to exploit the varying sign of the risk-return correlation. We explore the non-dominated frontier set up by portfolios with 12 or fewer stocks across the plane where:

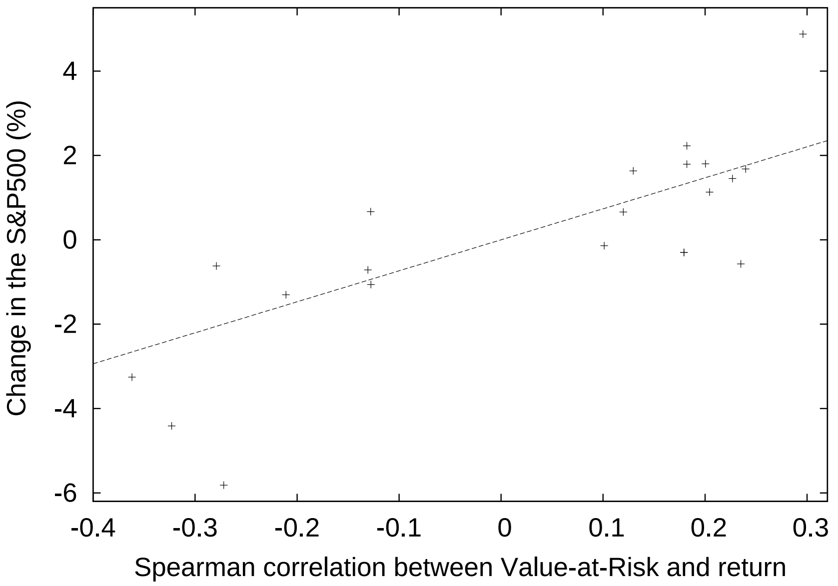

Fig. 3. For a sample of two-week periods in 2004-2007, about two-thirds of those periods have a Spearman's rank correlation significantly different from zero (at 95% confidence). For those periods, this shows the "flight to quality": when the S&P 500 rises, the risk-return correlation is positive; but when it declines, the risk-return correlation is negative.

- The X-axis is Value-at-risk (VaR) of the worst 5% of days, by Monte Carlo simulation of 50,000 trading days using a covariance matrix from doing Cholesky decomposition over the past 130 trading days.
- The Y-axis is the *reduction* in VaR between (a) the portfolio, and (b) the weighted average of the individual stocks of the portfolio. The greater the diversification, the greater the reduction in VaR.

Figure 4 shows this plane for 500 12-stock portfolios chosen from the S&P 500 on 15 February 2008. Naturally, we want a portfolio that is itself not volatile (towards the left) and is well diversified (towards the top). A line shows the non-dominated portfolios that form the frontier. The criteria in Section 3 chooses the portfolio towards the left side of that frontier in Figure 4, i.e., a highly-diversified portfolio with low volatility.

Our approach is unlike Markowitz,[3] who tries to explore the frontier of volatility and *expected return*. The notion of expected return can imply an opinion about the future. Instead, this paper tries instead to use something we can calculate today without any prejudices other than Figure 3.

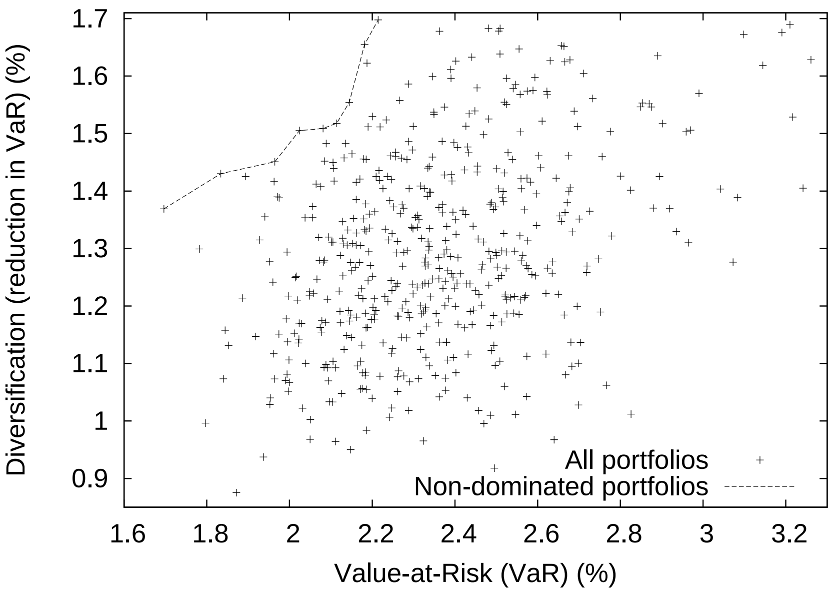

Fig. 4. For 500 12-stock portfolios on 15 February 2008, this shows the portfolio's VaR and the *difference* between the portfolio's VaR and the weighted average VaR of its constituent stocks. An investor who wants maximally-diversified, low-volatility portfolios should look at the frontier on the upper left.

2.1. *The Hillclimber*

A review of several optimization methods concluded that for evolutionary computation, "Only when some kind of local search is embedded at a second level, the computational results are reasonable ".[4] This paper uses evolution with a hill-climber to try and maximize diversification (the difference between a portfolio's VaR and the weighted average VaR of its constituent stocks) while aiming to keep the portfolio's VaR within a desired range (here less than 1.7%, the left side in Figure 4 above).

The hill-climber works by starting with a large number of stocks (here, a random number between 45 and 60 stocks), and then trying every possible portfolio with one less stock; that is, we remove each stock in turn from the portfolio and recalculate its VaR. Starting from an n-stock portfolio, there are n possible portfolios with $n-1$ stocks. We choose the portfolio that increases diversification the most while keeping the VaR closest to the desired range (here, less than 1.7%). This repeats, until the portfolio is down to only 12 stocks.

On these steps from a large to a small portfolio, there is a small probability of adding a stock; occasionally, it will randomly choose about 20 stocks, and for each one see if adding one more to the portfolio will improve diversification. This goes from n-stocks to $n+1$ stocks, and if any addition makes an improvement, then it keeps it.

Figure 4 shows that it is difficult to find a portfolio with such a low VaR. Sometimes this hill-climbing process ends with a 12-stock portfolio whose VaR is bigger than our desired 1.7%, as shown in Table 1.

2.2. *The Evolving Population of Portfolios*

Each call of the hill-climber produces a portfolio. We store these portfolios as a population in a genetic algorithm. To start we randomly choose between 45 and 60 stocks from the S&P 500 index, and use the hill-climber to whittle them down to only 12 stocks.

For crossover, we merely take two 12-stock portfolios and lump them together, and randomly add extra stocks to bring the total to somewhere between 45 and 60. Then the hill-climber whittles it down again.

For mutation, we simply take a 12-stock portfolio and again add some random stocks to bring the total to somewhere between 45 and 60.

This initial number of stocks (45 to 60) was chosen as a compromise to reduce run times. As the cardinality-limited portfolio optimization problem is NP-complete[2] there is no end to the computer time it can soak up.

These operators continue, selecting from the growing pool of 12-stock portfolios, until we have evaluated 500 portfolios; hence there are 500 points in Figure 4. Arguably, 500 samples may be too small for this problem. However, for some runs the hill-climber would produce the same final portfolio on the last few dozen attemps. This may be premature convergence, but it could indicate that there is no better solution to be found.

Hand-tuning on older data (2005-2007), suggested the following criteria:

- We take the non-dominated 12-stock portfolio whose VaR is between 1.63% and 1.65%; if none are in this range, take the one closest to 1.65%.
- Only take a 12-stock portfolio, or else take the largest portfolio; again, split ties by choosing the portfolio with VaR closest to 1.65%.

3. Results and Discussion

For all 19 two-week periods from 1 February to 24 October 2008, we use the above criteria for selecting a portfolio. Results are in Table 1.

Table 1. Returns for maximally diversified portfolios over 2-week periods.

Start date	12-stock portfolio Change	at VaR	S&P 500 index Start	End	Change	Ours is better?
2008-Feb-01	0.057%	1.858%	1395.42	1349.99	-3.256%	TRUE
2008-Feb-15	-0.582%	1.696%	1349.99	1367.68	1.310%	FALSE
2008-Feb-28	-4.807%	1.706%	1367.68	1288.14	-5.816%	TRUE
2008-Mar-14	0.537%	1.740%	1288.14	1315.22	2.102%	FALSE
2008-Mar-28	1.897%	1.788%	1315.22	1332.83	1.339%	TRUE
2008-Apr-11	5.847%	1.905%	1332.83	1397.84	4.878%	TRUE
2008-Apr-25	2.252%	1.714%	1397.84	1388.28	-0.684%	TRUE
2008-May-09	0.644%	1.763%	1388.28	1375.93	-0.890%	TRUE
2008-May-23	-0.366%	1.730%	1375.93	1360.68	-1.108%	TRUE
2008-Jun-06	-2.299%	1.717%	1360.68	1317.93	-3.142%	TRUE
2008-Jun-20	-5.634%	1.754%	1317.93	1262.90	-4.175%	FALSE
2008-Jul-03	2.306%	1.694%	1262.90	1260.68	-0.176%	TRUE
2008-Jul-18	3.029%	1.650%	1260.68	1260.31	-0.029%	TRUE
2008-Aug-01	3.906%	1.674%	1260.31	1298.20	3.006%	TRUE
2008-Aug-15	-2.738%	1.692%	1298.20	1282.83	-1.184%	FALSE
2008-Aug-29	-4.622%	1.690%	1282.83	1251.70	-2.427%	FALSE
2008-Sep-12	-1.259%	1.720%	1251.70	1213.27	-3.070%	TRUE
2008-Sep-26	-23.826%	1.653%	1213.27	899.22	-25.885%	TRUE
2008-Oct-10	-1.468%	2.119%	899.22	876.77	-2.497%	TRUE

Note: Periods start and end with Friday's closing prices (or Thursday, for holidays).

Table 1 gives impressive results. Binomially, it is significant at the 95% confidence level, which validates the approach. Of course, this can only work when risk and return have some relationship, and as discussed previously the relationship is absent in many 2-week periods where there is neither panic nor euphoria. The test periods above are in 2008 which happens to be a time where there is plenty of both panic and euphoria.

In conclusion, this strategy can work in either good or bad times, when even over the short term, the risk-return relationship exists (weak though it may be). But in more mundane times when the short-term risk-return relationship is absent, it may be less likely to give above-average returns.

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