OPTIMIZED TRADING AGENTS IN A TWO-STOCK PORTFOLIO USING MEAN-VARIANCE ANALYSIS*

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Based on the locality assumption and the mean-variance analysis for the resource allocation in a two-stock portfolio, we design a system of optimized trading agents which act according to the theoretical suggestion whenever a critical threshold of the Sharpe ratio is surpassed. The agents represent specific trading strategies that are characterized by four parameters that can be tuned via an adaptive online learning setting. When the theoretical suggestion is adopted, the resource allocation will be initiated with a fine tuning execution factor that represents the level of commitment of the agent to the suggestion. Using the buy-and-hold strategy as the benchmark, this system of agents have statistically outperformed the benchmark for the various two-stock portfolio taken from the Hang Sang Index in terms of higher and more stable profits.

1. Introduction

One of the interesting questions on portfolio management is to build a portfolio that has high return with low risk, and maintain this performance consistently over extended period. In this paper, we begin with the simplest portfolio of two stocks and apply the mean-variance analysis of Markowitz [1] to dynamically monitor the performance of the portfolio with real data taken from Hang Sang Index. Although the theory of mean variance analysis has been around for more than half a century, the dynamical implementation of the theory within the framework of multi-agent systems remains in its infancy. It is the

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objective of this paper to build a collection of agents, each equipped with an investment strategy based on the mean-variance analysis, and use the principle of the survivor of the fittest to evolve these agents in order to achieve the goal of high return, low risk and consistent performance [2-4].

One of the key assumptions we made concerns the locality of the stock daily data, in the sense that the return and volatility of stocks in the short period in the immediate past is assumed to be approximately followed [5,6]. Therefore, we do not make any attempt in forecasting the return and volatility. Our focus is on the competition of agents who all employ similar tools and have access to the same data base of past data. The agents are represented by strategies, which are characterized by parameters used in the portfolio management. With the locality assumption, daily trading strategies on the asset distribution among the portfolio are suggested by our approach. We compare the performance of the best strategy with a benchmark strategy, which is to buy the two stocks and hold on to them without trading over the entire period of the test. Our tests indicate that the performance of our collection of agents using simple-minded mean-variance analysis statistically outperform the buy-and-hold strategy. In some portfolios, the best agent can outperform with more than twice the return. In section 2, we describe the theory behind the formation of the agent. In section 3, we describe the rules we adopted for the evolution of the agents. We then test our multi-agent systems on data taken from combination of stocks in the Hang Sang Index.

2. Trading strategies

2.1. *Mean-variance theory*

In mean-variance analysis for two-stock portfolios, we need first to estimate the expected return U and variance Var of the stocks which are both parameterized by the sample size of the past data,

$$U(t) = \frac{1}{SampleSize} \sum_{k=t-SampleSize+1}^{t} r(k) \tag{1}$$

$$Var(t) = \frac{1}{SampleSize-1} \sum_{k=t-SampleSize+1}^{t} (r(k) - U(t))^2 \tag{2}$$

where $r(t) = \dfrac{p(t) - p(t-1)}{p(t-1)}$ is the daily rate of return and *p(t)* is the daily closing price of the stock. The standard deviation is $S(t) = \sqrt{Var(t)}$. The sample size used is the first parameter in our model.

We now design a portfolio consistent with *x* fraction of total investment in allocated to stock 1 and *1-x* invested in stock 2. We make the key assumption about locality in that the values of the rate of return within a local sampling subsequence can be specified by the same distribution. Thus U(t) is the estimation of *r(t+1)=u*. For two stocks, we can present a particular assignment of our portfolio as a point labeled by *x* in the mean-SD plane of the portfolio. For *x=1* the point corresponds to $(u,s)_{x=1} = \left(u_1(t), s_1(t)\right)$ and for *x=0*, we have $(u,s)_{x=0} = \left(u_2(t), s_2(t)\right)$. For general x, we have the portfolio mean and variance given by

$$u(t) \equiv u_1 x_1 + u_2 x_2 \tag{3}$$

$$Var(t) = Var_1 x_1^2 + Var_2 x_2^2 + 2Cov_{1,2} x_1 x_2 \tag{4}$$

where $x_1 = x$, $x_2 = 1 - x$ and $s(t) = \sqrt{Var(t)}$. Therefore, the portfolio at time *t* is described by the point $\left(u(t,x), s(t,x)\right)$ on the mean-SD plane. At any given time, we can find the value of x between 0 and 1 so that $F(t,x) = {u(t,x)}\big/{s(t,x)}$ is maximized. Let's denote the maximum by

$$\gamma^o(t) \equiv F^{\max}(t, x^o) = \max\left\{F(t,x) \mid 0 \le x \le 1\right\} \tag{5}$$

and the corresponding expected return and standard deviation by $\left(u^o(t), s^o(t)\right)$.

2.2. *Activation of Trading by a Sharpe ratio threshold*

We now propose a trading strategy adopted by the agent as follows,

- On each trading day *t*, those agents with $\gamma^o(t) > \gamma_c$ will suggest a modification of the existing portfolio to follow the suggested resource allocation change $x(t) \Rightarrow x(t+1) = x^o(t) \quad if \quad \gamma^o(t) > \gamma_c$
- On each trading day *t*, those agents with $\gamma^o(t) \le \gamma_c$ will sell all stocks and hold only cash.

In this activation model, we introduce a second parameter γ_c in our model.

2.3. *Execution factor*

According to the suggestion in 2.2, the portfolio of an agent will be updated. If we describe the portfolio of an agent by a vector $\vec{A}(t) \equiv \left(A_0(t), A_1(t), A_2(t)\right)$ where $A_0(t)$ is the percentage of wealth corresponding to the holding in cash, $A_1(t)$ is the percentage of wealth invested in stock 1, and $A_2(t)$ is the percentage of wealth invested in stock 2. Obviously we must have $\sum_{i=0}^{2} A_i(t) = 1$ as the normalization condition. Similarly, we can describe the suggestion made in our activation model by a vector $\vec{B}(t) \equiv \left(B_o(t), B_1(t), B_2(t)\right)$ where $B_o(t)$ is the suggested percentage of wealth held in cash, $B_1(t)$ and $B_2(t)$ are the suggested percentage of holding in stock 1 and 2. Note that we also have $\sum_{i=0}^{2} B_i(t) = 1$

Now we introduce an execution factor E for the agent. This factor is a measure of how much trust the agent has on the result of the mean-variance analysis and the activation mechanism. The agent's action will be defined by the following: $\vec{\alpha}(t, E) \equiv \left(\vec{A}(t) - \vec{B}(t)\right) \times E$ .This vector $\vec{\alpha}(t, E)$ is the vector defining the action the agent will take to align his existing portfolio vector $\vec{A}(t)$ with the suggestion $\vec{B}(t)$ by mean-variance analysis by proper buying and selling of the stocks. His commitment to follow the suggestion is characterized by the execution factor which is a number between 0 and 1. The agent's portfolio is updated by

$$\vec{A}(t+1) = \vec{A}(t) + \vec{\alpha}(t, E) \tag{6}$$

The execution factor E is our third parameter.

3. Trading agents in adaptive online learning

In section 2, we focus on the description of our agent in terms of its action and updating rule for its portfolio. There are three parameters introduced thus far: the sample size (λ_1) for measuring the mean return and variance, the critical threshold value of Sharpe ratio ($\lambda_2 = \gamma_c$), and the execution factor ($\lambda_3 = E$). Each agent can in principle adopt different set of parameters: $\vec{\lambda} = \left(\lambda_1, \lambda_2, \lambda_3\right)$. We now set up a multi-agent systems consisting a set of N agents described by a corresponding set of parameters vectors: $\{\vec{\lambda}(j,t) \mid j = 1,..,N\}$. Note that we explicitly include a time dependence on the parameter vector since we expect that the agents will evolve with time. This evolution of parameters is necessary in view of our application of the locality assumption for a small sample size, since in general there is a trend that the mean and variance will evolve in a longer time frame.

We now institute the competition among the agents, in the sense that we define a period T for the performance evaluation of these N agents and select the best performing agent (in terms of return of its portfolio net asset values at the end of the evaluation period). As we move along the time series of the stock market, we find out that the best agent is different at different time period, so that defines a trajectory of the best agent parameter vector $\vec{\lambda}^*(t)$ in parameter space. One then has an elementary scheme for adaptive online learning to follow the best agent.

In this paper, we avoid a complex online learning algorithm for the agents, which can easily modeled by genetic algorithm. We simply pre-define a set of N vectors $\{\vec{\lambda}(j) \mid j=1,...,N\}$ at the beginning and ensure that these N vectors are distributed evenly in the three-dimensional parameter space. At each time step, we locate the best performing agent by its index. Thus, if the best performing agent is *k* at *t*, then we use $\vec{\lambda}^* = \vec{\lambda}(k)$. Our scheme is based on the simplest rule of following the leader.

Recall that our evaluation of the N agents is over a period T. On each trading day, the performances of agents in the past period T are reviewed and the parameters of the best agent are copied as the current parameters for the strategy on this trading day. Therefore, in principle, we have a fourth parameter in our model, which is the period of performance evaluation.

4. Results

We apply the above trading strategies to the Hang Seng Index (HSI) constituent stocks. The risk-free rate of return is assumed to be zero, which means the stock market will be our only investment instrument. The parameters we used are in the range: $E \in \{0.1, 0.2, ..., 1.0\}$, sample size in the range {20, 22,.., 30}, and $\gamma_c \in \{-2.5 + 0.12 * i \mid i = 0, 1, 2, .., 29\}$. For HSI, there are 23 stocks which have been in HSI constituent list for the recent 8 years. For a two-stock portfolio, we have thus a total of 253(=23*22/2) two-stock portfolios. We simulate our trading strategies for all these portfolios in 1000 trading days (from March 2004 to February 2008). We compare our “follow-the-leader” portfolio with a buy-and-hold (BNH) strategy, which reflects the prospect of intrinsic performance of the selected stocks. If we choose stock 1 and 2, then the rate of return for the BNH strategy is defined by $r_{BNH} = (r_{BNH,1} + r_{BNH,2})/2$. The rate of return by our portfolio management analysis is denoted as r_P . Both r_{BNH} and r_P for all the 253 portfolios are plotted in the Fig.1. Among all the 253 portfolios, there are 148 portfolios for which our trading strategy outscores BNH ($r_P > r_{BNH}$). The average of r_P among all the 253 portfolios is 0.309, while the average of r_{BNH} is 0.123. In this sense, our strategy is 151% better.

We should also emphasize the excellent performance of our portfolio when the buy-and-hold strategy yields negative return, (r_{BNH} <0). Among the 253 pairs of stocks there are 88 such pair yield negative r_{BNH} . However, among these 88 pairs, 75 pairs yield better return than the BNH strategy using our strategy. For these 88 pairs, the average for r_{BNH} is -0.194, while the average of r_P is 0.274 This observation indicates the advantage and robustness of our multi-agent systems approach in portfolio management. This observation has also been verified for the constituent stocks in Dow Jones Industrial Average (DJIA). There are 25 stocks which have been in DJIA constituent list for the recent 8 years. We can form a total of 300(=25*24/2) two-stock portfolios from DJIA. Among all the 300 pairs, there are 55 portfolios with negative r_{BNH} . For these 55 portfolios, 35 of them have $r_P > r_{BNH}$. For these 35 pairs, the average of r_{BNH} is -0.144, while the average of r_P is 0.013.

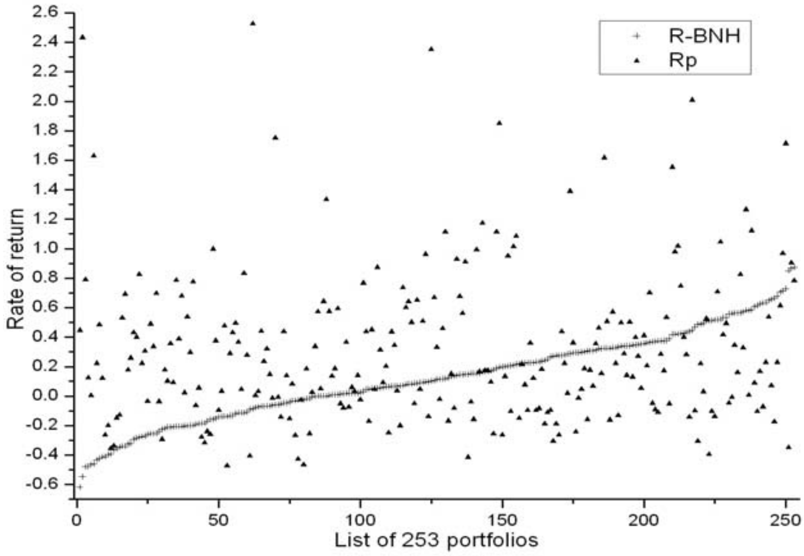

Figure 1. For HSI constituent stocks, the rate of return obtained by our “follow-the-leader” trading strategy is compared with the BNH rate of return. The x axis is ordered by the BNH rate of return of the 253 two-stock portfolios from HSI stocks. The cross is for the BNH strategy while the triangles are the “follow-the-leader” strategy.

5. Discussion

From our test on HSI, we can make two observations. The first observation concerns the distribution of points in Fig.1. We see that the rate of return obtained by our trading strategy is always distributed above -0.50 for all portfolios and sometime has a return as high as 2.5. We can interpret this as saying that the loss for our strategy is bounded by 50%, but the expected return can reach as high as 250%. The second observation concerns those portfolios with negative BNH rate of return. For these portfolios, our trading strategy leads to a better outcome for most of the cases. In HSI, not only does the average return over 253 portfolios is better then BNH, the return is usually positive even though the BNH rate of return of the portfolio is negative. This ability to squeeze profits from two decreasing stocks suggests that our algorithm for the follow-the-leader strategy in our multi-agent-systems presents valuable insight in the mean-variance analysis of portfolio, in that it performs well even in a bear market. Our future research will address the issue of extending our theory to include more than two stocks in the portfolio, as well as an intelligent evolution of online learning for our agents in the multi-dimensional parameter space [7,8].

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