THE ENTRY AND EXIT STRATEGIES FOR A SUPPLY CONTRACT UNDER STOCHASTIC MARKET DEMANDS: A REAL OPTIONS APPROACH

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This paper extends the real option model of Dixit (1989) to propose the valuation of a supply contract with order flexibility. We assume that there are 2 echelons in the supply chain system: the business market and the consumer market. A single-supplier and single-retailer system is considered. The demands in the consumer market evolve as a stochastic process, which follows a geometric Brownian motion (GBM) on the continuous time horizons. By using the dynamic programming approach, we derive a system of non-linear equations, which are in turn solved for the real option value and entry/exit thresholds by our proposed numerical algorithm, extended from the nonlinear least squares minimization algorithm by Levenberg (1944) and Marquardt (1963). The results of this paper will provide the decision makers with more insights into the interactions among the market dynamics and the option value, as well as the entry/exit thresholds. The conclusions will also help the practitioners reach better decisions on the entry and exit strategies for a supply contract when the market demands and price uncertainties are the major value drivers.

Keyword: Supply Contract, Stochastic Market Demand, Entry/Exit Strategy, Real Option

1. Introduction

In the practice of the coordination within a supply chain, the retailer may transfer the market risks (i.e. demands and price uncertainties) to the supplier via a supply contract, which allows the retailer to place an order at the predetermined wholesale price on or before the maturity date of this contract. In such a setting, the supplier should accumulate more inventories or shorten the lead time of goods-delivery to meet the retailer's needs. When a retailer has a right to lock in the wholesale price and to place orders flexibly, depending on the dynamic market demands, she owns a contractual real option. This paper addresses the pricing problems and then proposes a contractual real option model in the stochastic context.

Moreover, the retailer has to determine when to sign a supply contract to begin the goods-delivery from the supplier and when to quit it. We refer them as entry and exit strategies for the retailer in this paper. In our model, the underlying real asset is a supply contract, whose value is the discounted profits generated from it. The exercise price of the entry option is the contracting fees to activate the continuous goods-delivery from the supplier, while the exercise price of the exit option is the cancellation fees. Therefore, to maximize the contract value, two critical decisions need to be made for the retailer: (1) when should the retailer enter a contract to begin the continuous goods delivery from the supplier ? (2) when should the retailer cancel the on-going goods delivery to quickly respond to the declining market demands?

Concerning the problems addressed above, several scholars have developed models in the real option approach to discuss the best strategies for the entry and exit of investment projects. Pindyck (1991) indicated that waiting for investment is valuable when the underlying real asset, which generates the investment profits, is volatile and evolves stochastically. The traditional Net Present Value (NPV) approach advocates that any decision on investment should be made currently, which lacks of the waiting flexibility, and undervalues the investment opportunity. Conversely, Dixit (1989) proposed the real option model for the decisions on the optimal exit time to shut down a factory (waiting for divestment).

This paper extends the model of the combined entry/exit strategies proposed by Dixit (1989) and develop a further theoretical model to deal with the entry and exit decisions for a supply contract. In our model, the maximized contract value is obtained through the pricing of the real option, while the entry and exit strategies are suggested and based on the

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price thresholds (entry/exit thresholds) derived in our work. However, analytical solutions are not obtained in this paper because of the non-linear simultaneous equations developed by us. Instead, we propose a numerical algorithm to approximate the closed form solutions.

This paper is organized as follows: Section 1 discusses the entry and exit strategies in a real option approach, proposed by Pindyck, and the nonlinear least squares minimization algorithm referred to as Levenberg-Marquardt (LM) algorithm, proposed by Levenberg (1944) and Marquardt (1963). Section 2 will extend the combined entry and exit option model to the supply contract decisions faced with by the retailer in a single-supplier/single-retailer system. The optimized profits generated form the supply contract under the stochastic market demands are proposed. A system of non-linear simultaneous equations are obtained form the model. The LM optimization algorithm is extended to solve the simultaneous equations for the entry and exit thresholds and option values. In Section 3, we present the numerical results in tables and figures to show that the sensitivity of the entry/exit thresholds and the option values to the price elasticity and the volatility of the market demands. Section 4 concludes this paper and recommends the possible further studies.

2.The Model

2.1 Assumptions and Notations

To formulate this model, the unit cost of goods delivered from a supplier to a retailer at time t is denoted by C_t (wholesale price), while the unit price of goods sold to the final customers denoted by P_t (Retail Price). The quantities delivered and sold to the final customers at the same time t is denoted by Q_t, and the margin profit of the retailer through the supply contract denoted by π_t. The risk-free interest rate in the financial market is denoted by r_f, and the discount rate denoted by δ. We assume that the goods delivered to a retailer could be sold to the final customers immediately, and therefore there is no inventory in the supply chain. When the retailer enters a contract to activate the goods-delivery from the supplier, the delivery will continuously last until the cancellation by the retailer. Conversely, after the retailer cancels the contract, it will wait for the next opportunity to enter another supply contract. We also assume that the supply contract allow the retailer to receive goods from the supplier without maturity at a predetermined wholesale price, which is a constant proportion of the retailer price.

According to the model proposed by Carruth et al.(2000), the relationship between the retail price and the quantities delivered to the retailer and immediately sold to the customers at time t could be obtained through the inversed demand function as follows:

$$P_t = \phi_t Q_t^{-\frac{1}{\eta}}, \tag{1}$$

where η is the elasticity of the market demands, and ϕ_t is an uncertainty factor and referred to as demand shift, which is a geometric Brownian motion (GBM) process as follows:

$$d\phi_t = \phi_t(\alpha dt + \sigma dW_t), \tag{2}$$

where α is the instantaneous growth rate, σ is the volatility and dW_t is the increment of a Wiener process at time t. Under the real world probability measure, ϕ_t follows a lognormal distribution at time t with mean αt, and variance $\sigma^2 t$. That is, $\ln\phi_t \sim Normal(\ln\phi_0 + \alpha t, \sigma^2 t)$. The increment of W_t follows a normal distribution at time t and is independent of W_t. Moreover, dW_t and dW_s are non-overlapping $(s > t)$. That is, $dW_t \sim Normal(0, t)$.

Based on the model of Schwartz and Moon (2000), the unit cost of goods sold is proportional to the wholesale price at time t and could be expressed as follows:

$$C_t = (1-\gamma)P_t, \tag{3}$$

If we rearrange equation (3), it becomes $P_t - C_t = \gamma P_t$, where γ is a constant margin profit rate $(0 < \gamma < 1)$ and γP_t is retailer's unit margin profit.

We assume that the market demand follows a downward-sloping curve and is an exponential distribution on the time horizon as follows:

$$Q_t = \frac{1}{\theta}\exp\left[-\left(\frac{t-t_0}{\theta}\right)\right], \tag{4}$$

where $\theta > 0$, and t_0 is the beginning time of the product life cycle. Without loss of generality, we could assume that the product life cycle starts at $t_0 = 0$, and equation (4) could be reduced to

$$Q_t = \frac{1}{\theta}\exp\left(-\frac{t}{\theta}\right). \tag{5}$$

Combine equations (1), (3), and (4), we obtain the dynamics of the retailer's margin profits generated from the supply contract as follows:

$$\pi_t(\phi_t, t) = Q_t(P_t - C_t) = Q_t\gamma P_t = \gamma\phi_t Q_t^{-\frac{1}{\eta}} \cdot Q_t = \gamma\phi_t Q_t^{1-\frac{1}{\eta}}. \tag{6}$$

2.2 The formulation of the Model

If we apply the Ito's Lemma to equation (6), we could obtain the dynamics of π_t:

$$\frac{d\pi_t}{\pi_t} = [\alpha - (1-\frac{1}{\eta})\frac{1}{\theta}]dt + \sigma dW_t, \tag{7}$$

Where $\delta < \alpha - (1-\frac{1}{\eta})\frac{1}{\theta}$, otherwise, the retailer would rather wait than place any order to the supplier. Since π_t is a derived GBM process, it also follows a lognormal distribution with mean, $[\alpha - (1-\frac{1}{\eta})\frac{1}{\theta}]t$, and variance, $\sigma^2 t$. That is,

$$\ln \pi_t \sim Normal(\ln \pi_0 + [\alpha - (1-\frac{1}{\eta})\frac{1}{\theta}]t, \sigma^2 t).$$

We define V_t as the value of the supply contract at time t. Then we have

$$V_t = \frac{\pi_t}{\delta - \alpha + (1-\frac{1}{\eta})\frac{1}{\theta}}, \tag{8}$$

and the dynamics of V_t is

$$\frac{dV_t}{V_t} = \frac{d\pi_t}{\pi_t} = [\alpha - (1-\frac{1}{\eta})\frac{1}{\theta}]dt + \sigma dW_t. \tag{9}$$

Similarly, V_t evolves as a derived GBM, and has the same distribution as π_t does. That is,

$$\ln V_t \sim Normal(\ln V_0 + [\alpha - (1-\frac{1}{\eta})\frac{1}{\theta}]t, \sigma^2 t).$$

Let F denote the option value of the supply contract. By applying the Ito's Lemma, we could derive the dynamics of F as follows:

$$\begin{aligned} dF &= \frac{\partial F}{\partial t}dt + \frac{\partial F}{\partial V}dV + \frac{1}{2}\frac{\partial^2 F}{\partial V^2}(dV)^2 \\ &= \left\{\frac{1}{2}\sigma^2 V^2 F'' + [\alpha - (1-\frac{1}{\eta})\frac{1}{\theta}]VF'\right\}dt + \sigma VF' dW, \end{aligned} \tag{10}$$

We intend to maximize the discounted value of the supply contract with respect to t as follows:

$$F(V_t, t) = \max_t E_t[(V_t - I)e^{-rt}], \tag{11}$$

Where I denotes the contracting fees to activate the continuous goods-delivery from the supplier. We could suppress $F(V_t, t)$ as F. The Bellman equation is:

$$r_f F = \max_t[\pi_t + \frac{1}{dt}E_t(dF)] = \max_t\left\{\pi_t + \frac{1}{2}\sigma^2 V^2 F'' + [\alpha - (1-\frac{1}{\eta})\frac{1}{\theta}]VF'\right\}.$$

It implies that $\max_t\left\{\pi_t + \frac{1}{2}\sigma^2 V^2 F'' + [\alpha - (1-\frac{1}{\eta})\frac{1}{\theta}]VF' - r_f F\right\} = 0$. (12)

In order to solve for the values of the entry and exit options, we define two states for the supply contract. First one is the idle state in which the retailer waits for the best opportunity to activate the delivery of goods because of the less operational profits. Second one is the active state in which the retailer continuously receives from the supplier the quantities needed by the final consumers. The retailer then closely monitors the declining market demands to decide when to cancel the goods-delivery. We discuss the above two states in more details to obtain the option value and the entry/exit thresholds.

In the idle state, the supply contract will not generate any profit ($\pi_t = 0$) until the stochastic contract value, V_t , reaches the entry threshold, V_H .The time when equation (11) is maximized is referred to as the optimal entry time for the supply contract. We suppress $F_I(V_t, t)$ as F_I , and equation (12) becomes:

$$\frac{1}{2}\sigma^2 V^2 F_I'' + [\alpha - (1-\frac{1}{\eta})\frac{1}{\theta}]VF_I' - r_f F_I = 0 \quad . \tag{13}$$

The solution to equation (12) is assumed to be in the following form:

$$F_I(V_t, t) = A_1 V_t^{b_1} + A_2 V_t^{b_2} \quad . \tag{14}$$

Moreover, $F_I(V_t, t)$ must satisfy the following three boundary conditions:

$$F_I(0,t) = 0 \; . \tag{15}$$

$$F_I'(V_t, t) = A_1 b_1 V_t^{b1-1} + A_2 b_2 V_t^{b2-1} \; , \tag{16}$$

$$F_I''(V_t, t) = A_1 b_1 (b_1 - 1) V_t^{b1-2} + A_2 b_2 (b_2 - 1) V_t^{b2-2} \; , \tag{17}$$

If we substitute the above equations (14), (16), and (17) into equation (13), and rewrite it as:

$$\frac{1}{2}\sigma^2 V_t^2 \left[A_1 b_1 (b_1 - 1) V_t^{b_1-2} + A_2 b_2 (b_2 - 1) V_t^{b_2-2} \right] + [\alpha - (1-\frac{1}{\eta})]\frac{1}{\theta}]V_t \left(A_1 b_1 V_t^{b_1-1} + A_2 b_2 V_t^{b_2-1} \right)$$
$$-r_f \left(A_1 V_t^{b_1} + A_2 V_t^{b_2} \right) = 0,$$

we can obtain:

$$\left\{ \frac{1}{2}\sigma^2 b_1 (b_1 - 1) + [\alpha - (1-\frac{1}{\eta})\frac{1}{\theta}]b_1 - r_f \right\} A_1 V_t^{b_1} + \left\{ \frac{1}{2}\sigma^2 b_2 (b_2 - 1) + [\alpha - (1-\frac{1}{\eta})\frac{1}{\theta}]b_2 - r_f \right\} A_2 V_t^{b_2} = 0 \; .$$

Since $A_i V_t^{b_i} \neq 0, where\; i = 1,2$, we have:

$$\frac{1}{2}\sigma^2 b_1 (b_1 - 1) + [\alpha - (1-\frac{1}{\eta})\frac{1}{\theta}]b_1 - r_f = 0 \; , \tag{18}$$

and $\frac{1}{2}\sigma^2 b_2 (b_2 - 1) + [\alpha - (1-\frac{1}{\eta})\frac{1}{\theta}]b_2 - r_f = 0 \; .$ (19)

We can combine equations (18) and (19) to rewrite

$$\frac{1}{2}\sigma^2 b(b-1) + [\alpha - (1-\frac{1}{\eta})\frac{1}{\theta}]b - r_f = 0 \; , \tag{20}$$

And then solve for equation (20). Let b_1 and b_2 denote the solutions to the above quadratic equation, and we can obtain:

$$b_1 = \frac{1}{2} - \frac{[\alpha - (1-\frac{1}{\eta})\frac{1}{\theta}]}{\sigma^2} + \left\{ \left(\frac{1}{2} - \frac{[\alpha - (1-\frac{1}{\eta})\frac{1}{\theta}]}{\sigma^2} \right)^2 + \frac{2r_f}{\sigma^2} \right\}^{\frac{1}{2}} > 0 \text{ ,and } b_2 = \frac{1}{2} - \frac{[\alpha - (1-\frac{1}{\eta})\frac{1}{\theta}]}{\sigma^2} - \left\{ \left(\frac{1}{2} - \frac{[\alpha - (1-\frac{1}{\eta})\frac{1}{\theta}]}{\sigma^2} \right)^2 + \frac{2r_f}{\sigma^2} \right\}^{\frac{1}{2}} < 0 \; .$$

Since b_2 is negative, and $F_I(V_t, t)$ satisfies the first boundary condition $F_I(0,t) = 0$, we can conclude that $A_2 = 0$. We finally obtain $F_I(V_t, t) = A_1 V_t^{b_1}$. (21)

Similarly, when the contract is in the active state, we denote the option value as $F_A(V_t, t)$ and suppress it as F_A . The profit flow generated from the supply contract at time t is

$$\pi_t = \left[\delta - \alpha + (1-\frac{1}{\eta})\frac{1}{\theta} \right] V_t \; .$$

Therefore, equation (12) becomes:

$$\max_t \left\{ \left[\delta - \alpha + (1-\frac{1}{\eta})\frac{1}{\theta} \right] V + \frac{1}{2}\sigma^2 V^2 F_A'' + [\alpha - (1-\frac{1}{\eta})\frac{1}{\theta}] V F_A' - r_f F_A \right\} = 0 \tag{22}$$

After the optimal entry time t^* has been found, equation (21) becomes

$$\frac{1}{2}\sigma^2 V^2 F_A'' + [\alpha - (1-\frac{1}{\eta})\frac{1}{\theta}] V F_A' - r_f F_A + \left[\delta - \alpha + (1-\frac{1}{\eta})\frac{1}{\theta} \right] V = 0 \,, \tag{23}$$

The solution to equation (23) is assumed to be in the following form :

$$F_A(V_t, t) = B_1 V_t^{b_1} + B_2 V_t^{b_2} + \left[\frac{\alpha + (1-\frac{1}{\eta})\frac{1}{\theta} - \delta}{\alpha - (1-\frac{1}{\eta})\frac{1}{\theta} - r_f} \right] V_t \,, \tag{24}$$

where b_1 and b_2 are the same as the above definition (proved in Appendix B).

Moreover, $F_A(V_t, t)$ must satisfy the boundary condition: $\lim_{V_t \to \infty} \frac{F_A(V_t,t)}{V_t} < \infty$. (25)

Therefore, b_1 should be greater than 1 and $B_1 = 0$. Equation (23) becomes

$$F_A(V_t, t) = B_2 V_t^{b_2} + \left[\frac{\alpha + (1-\frac{1}{\eta})\frac{1}{\theta} - \delta}{\alpha - (1-\frac{1}{\eta})\frac{1}{\theta} - r_f} \right] V_t \,. \tag{26}$$

Let $k = \dfrac{\alpha + (1-\frac{1}{\eta})\frac{1}{\theta} - \delta}{\alpha - (1-\frac{1}{\eta})\frac{1}{\theta} - r_f}$, then $F_A(V_t, t) = B_2 V_t^{b_2} + k V_t$. (27)

The boundary conditions are as follows:

$$F_I(V_H, t) = F_A(V_H, t) - I \,, \tag{28}$$

$$F_I'(V_H, t) = F_A'(V_H, t) \,, \tag{29}$$

$$F_A(V_L, t) = F_I(V_L, t) - E \,, \tag{30}$$

$$F_A'(V_L, t) = F_I'(V_L, t) \,, \tag{31}$$

In equations (28) and (30), I and E denote contracting and cancellation fees, respectively. Equation (28) says that the retailer will pay the contracting fees to exercise the entry option on the entry threshold level to enter a supply contract and simultaneously obtain the exit option. We can substitute equations (21) and (26) into Equations (28), (29), (30), and (31). The following system of equations can be obtained:

$$-A_1 {V_H}^{b_1} + B_2 {V_H}^{b_2} + k V_H = I \,, \tag{32}$$

$$-b_1 A_1 {V_H}^{b_1 - 1} + b_2 B_2 {V_H}^{b_2 - 1} + k = 0 \,, \tag{33}$$

$$-A_1 {V_L}^{b_1} + B_2 {V_L}^{b_2} + k V_L = -E \,, \tag{34}$$

$$-b_1 A_1 {V_L}^{b_1 - 1} + b_2 B_2 {V_L}^{b_2 - 1} + k = 0 \,, \tag{35}$$

2.3 The Algorithm to Solve for Option Values and Entry/Exit Thresholds

To solve for the above non-linear system of equations, we develop the following algorithm, which is extended from the Levenberg-Marquardt (LM) nonlinear least squared minimization algorithm proposed by Levenberg(1944) and Marquardt (1963), to obtain the numerical solutions for the entry threshold (V_H), exit threshold (V_L), entry option value $F_I(V)$, and exit option value $F_A(V)$.

Numerical Algorithm to Solve for Equations (32) through (35) :

Step 1: We define

$$f_1 = -A_1 {V_H}^{b_1} + B_2 {V_H}^{b_2} + k V_H - I \,,$$

$$f_2 = -b_1 A_1 V_H^{\,b_1 - 1} + b_2 B_2 V_H^{\,b_2 - 1} + k \text{ ,}$$

$$f_3 = -A_1 V_L^{\,b_1} + B_2 V_L^{\,b_2} + k V_L + E \text{ ,}$$

and $f_4 = -b_1 A_1 V_L^{\,b_1 - 1} + b_2 B_2 V_L^{\,b_2 - 1} + k$, where $k = (\alpha + (1 - \frac{1}{\eta})\frac{1}{\theta} - \delta) \div (\alpha - (1 - \frac{1}{\eta})\frac{1}{\theta} - r_f)$.

We also define $x = (A_1, B_2, V_H, V_L)$.

Given $x_0 = (A_1^0, B_2^0, V_H^0, V_L^0)$, let $x = x_0$ as the Initial Guess Number.

Define $g(x) = \sum_{i=1}^{4} f_i^2(x)$ as the sum of squared estimation errors.

Choose a small positive number, ε , and set $\lambda = 1$

Step 2: Compute the Gradient of g, denoted by ∇g . If $\|\nabla g\|_\infty < \varepsilon$ (Stopping Criteria), stop the algorithm and x is the estimated solution set with minimum sum of squared estimation errors. Otherwise, proceed to step 3 for further estimation steps.

Step 3: Let old_g = $g(x)$, and compute the Hessian matrix of $g(x)$, denoted by $H = \left[\frac{\partial^2 r}{\partial x_i \partial x_j} \right]_{i,j}$.

Step 4: Compute $M = H + \lambda \cdot diag(H)$.

Step 5: Solve $M \cdot z = \nabla g$ for z .

Step 6: Set new_ $x = x - z$ and new_g= $g(new\_x)$.

Step 7: If new_g < old_g, set λ to 0.1λ and x to new_ x . Otherwise, if new_g $\geq$ old_g, set λ to 10λ , return to Step 2 until the stopping criteria has been satisfied.

3. The Numerical Results

The numerical results, obtained form the algorithm in the previous section, are shown in Tables 3.1 through 3.4. It can be shown that when the volatility of demand shock increases from 0.315 to 0.335 and the price elasticity of demand and the discount rate are set to 1.015 and 0.001 respectively, the entry threshold increase from 80.4 to 86.7 in Table 3.1, while the corresponding exit threshold also increases from 60.6 to 64.8 in table 3.2. Similarly, in tables 3.3 and 3.4, the corresponding entry and exit option values also increase from 11.1 and 116.3 to 11.3 and 116.6, respectively. When we vary the discount rate from 0.001 through 0.005, the above positive correlations are still applicable.

According to the graphics drawn from the numerical results, the entry and exit option values are more sensitive to the price elasticity of the demand than to the demand volatility as shown in figure 3.1 and figure 3.2. The price elasticity of demand and the volatility of demand shock have positive impacts on both the option values and the value of the thresholds. That is, the greater volatility and the greater price elasticity of demand contribute to the greater entry/exit option values and to the higher entry/exit thresholds.

Table 3.1 The Sensitivity of Entry Threshold (V_H) under the parameter set $V = 100, \theta = 2, \alpha$=0.1

V_H		$\delta = 0.001$					$\delta = 0.002$					$\delta = 0.003$					$\delta = 0.004$					$\delta = 0.005$				
		σ					σ					σ					σ					σ				
		0.315	**0.320**	**0.325**	**0.330**	**0.335**	**0.315**	**0.320**	**0.325**	**0.330**	**0.335**	**0.315**	**0.320**	**0.325**	**0.330**	**0.335**	**0.315**	**0.320**	**0.325**	**0.330**	**0.335**	**0.315**	**0.320**	**0.325**	**0.330**	**0.335**
η	**1.015**	80.4	81.8	82.3	85.6	86.7	80.6	82.5	82.5	85.6	86.6	79.0	82.9	83.6	86.2	86.8	80.7	81.6	85.9	86.7	87.5	81.1	83.0	83.1	86.5	90.4
	1.020	81.0	82.3	84.7	85.7	85.2	80.9	81.9	84.9	86.0	85.5	81.5	83.0	85.1	85.9	86.3	80.2	83.8	83.4	86.5	86.7	81.2	82.8	83.4	88.5	90.5
	1.025	80.4	83.8	85.1	84.9	83.7	80.9	81.1	85.1	85.0	84.4	81.6	83.0	85.2	85.7	84.8	82.6	83.0	85.5	86.6	88.4	80.8	85.4	86.1	85.5	92.7
	1.030	80.5	84.1	84.3	83.5	86.7	81.3	84.2	85.4	84.0	87.4	81.8	84.3	85.5	84.7	88.4	81.3	84.5	85.4	85.3	87.3	82.9	84.8	85.9	85.4	93.8
	1.035	83.5	85.1	83.2	81.1	85.1	83.3	84.4	83.7	82.2	86.4	83.4	84.4	83.7	82.8	87.4	83.5	84.5	84.8	79.9	86.4	81.5	85.0	88.2	84.8	92.4

Table 3.2 The Sensitivity of Exit Threshold (V_L) under the parameter set $V = 100, \theta = 2, \alpha$=0.1

V_L		$\delta = 0.001$					$\delta = 0.002$					$\delta = 0.003$					$\delta = 0.004$					$\delta = 0.005$				
		σ					σ					σ					σ					σ				
		0.315	**0.320**	**0.325**	**0.330**	**0.335**	**0.315**	**0.320**	**0.325**	**0.330**	**0.335**	**0.315**	**0.320**	**0.325**	**0.330**	**0.335**	**0.315**	**0.320**	**0.325**	**0.330**	**0.335**	**0.315**	**0.320**	**0.325**	**0.330**	**0.335**
η	**1.015**	60.6	61.5	61.8	64.3	64.8	60.6	62.0	61.6	64.3	64.7	59.2	62.3	62.6	64.5	64.9	60.5	61.0	64.4	64.8	65.3	60.7	62.1	61.9	64.7	67.7
	1.020	61.3	62.1	64.0	64.6	63.8	61.1	61.6	63.9	64.8	64.1	61.6	62.5	64.1	64.6	64.6	60.2	63.1	62.6	65.0	64.7	61.1	62.3	62.5	66.7	68.1
	1.025	61.0	63.8	64.7	64.0	62.9	61.4	61.3	64.5	64.2	63.4	61.9	62.8	64.4	64.6	63.6	62.5	62.8	64.6	65.3	66.6	61.0	64.7	65.0	64.2	70.3
	1.030	61.3	64.1	64.1	63.3	65.7	61.9	64.3	65.0	63.6	66.3	62.1	64.1	64.8	63.9	66.9	61.8	64.3	64.9	64.4	66.0	63.2	64.6	65.1	64.6	71.5
	1.035	64.2	65.3	63.6	61.3	64.4	64.0	64.8	63.9	62.2	65.5	63.8	64.6	63.6	62.7	66.3	63.9	64.6	64.5	60.1	65.3	62.1	64.8	67.3	64.3	70.4

Table 3.3 The Sensitivity of Entry option (F_I) under the parameter set $V = 100, \theta = 2, \alpha = 0.1$

F_I		δ = 0.001					δ = 0.002					δ = 0.003					δ = 0.004					δ = 0.005				
		σ					σ					σ					σ					σ				
		0.315	**0.320**	**0.325**	**0.330**	**0.335**	**0.315**	**0.320**	**0.325**	**0.330**	**0.335**	**0.315**	**0.320**	**0.325**	**0.330**	**0.335**	**0.315**	**0.320**	**0.325**	**0.330**	**0.335**	**0.315**	**0.320**	**0.325**	**0.330**	**0.335**
	1.015	11.1	11.1	11.2	11.2	11.3	12.1	12.2	12.3	12.4	12.5	13.3	13.5	13.6	13.8	13.9	14.6	14.8	15.0	15.2	15.4	16.0	16.2	16.4	16.6	16.9
	1.020	11.2	11.2	11.3	11.3	11.4	12.3	12.4	12.5	12.6	12.7	13.6	13.7	13.8	14.0	14.2	14.9	15.1	15.3	15.5	15.7	16.3	16.6	16.8	17.1	17.4
η	**1.025**	11.2	11.3	11.3	11.4	11.4	12.4	12.5	12.6	12.8	12.9	13.8	13.9	14.1	14.3	14.5	15.2	15.4	15.6	15.9	16.1	16.7	17.0	17.3	17.6	17.9
	1.030	11.3	11.4	11.4	11.5	11.5	12.6	12.7	12.8	13.0	13.1	14.0	14.2	14.4	14.6	14.8	15.6	15.8	16.0	16.3	16.6	17.2	17.5	17.8	18.1	18.5
	1.035	11.4	11.5	11.6	11.6	11.6	12.8	12.9	13.0	13.2	13.4	14.3	14.5	14.7	15.0	15.2	16.0	16.2	16.5	16.8	17.1	17.7	18.0	18.3	18.7	19.1

Table 3.4 The Sensitivity of Exit option (F_A) under the parameter set $V = 100, \theta = 2, \alpha = 0.1$

F_A		δ = 0.001					δ = 0.002					δ = 0.003					δ = 0.004					δ = 0.005				
		σ					σ					σ					σ					σ				
		0.315	**0.320**	**0.325**	**0.330**	**0.335**	**0.315**	**0.320**	**0.325**	**0.330**	**0.335**	**0.315**	**0.320**	**0.325**	**0.330**	**0.335**	**0.315**	**0.320**	**0.325**	**0.330**	**0.335**	**0.315**	**0.320**	**0.325**	**0.330**	**0.335**
	1.015	116.3	116.3	116.4	116.5	116.6	116.5	116.5	116.6	116.6	116.7	116.6	116.7	116.7	116.8	116.8	116.8	116.8	116.9	116.9	117.0	117.0	117.0	117.1	117.1	117.2
	1.020	122.2	122.3	122.3	122.4	122.5	122.4	122.5	122.5	122.6	122.7	122.7	122.7	122.8	122.8	122.9	122.9	122.9	123.0	123.1	123.1	123.2	123.2	123.2	123.3	123.4
η	**1.025**	128.4	128.4	128.5	128.6	128.7	128.7	128.7	128.8	128.9	129.0	129.0	129.0	129.1	129.2	129.3	129.3	129.3	129.4	129.5	129.6	129.6	129.7	129.7	129.8	129.9
	1.030	134.8	134.9	135.0	135.1	135.3	135.2	135.3	135.4	135.5	135.6	135.6	135.6	135.7	135.8	135.9	136.0	136.1	136.1	136.2	136.3	136.4	136.5	136.5	136.6	136.7
	1.035	141.6	141.7	141.8	142.0	142.1	142.0	142.1	142.2	142.4	142.5	142.5	142.6	142.7	142.8	142.9	143.0	143.1	143.2	143.3	143.4	143.6	143.6	143.7	143.8	143.9

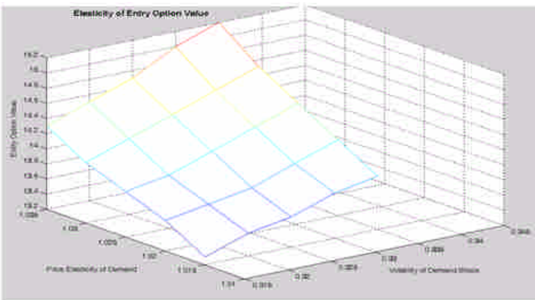

Fig.3.1 The Sensitivity of the Entry Option Value under the parameter set: V=100, θ =2.0, α =0.1, δ =0.003

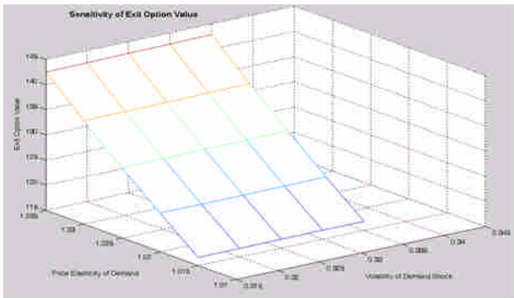

Fig.3.2 The Sensitivity of the Exit Option Value under the parameter set: V=100, θ =2.0, α =0.1, δ =0.003

4. Conclusions

A real option model is employed in this paper to investigate the relationships among the option values, the entry/exit thresholds and the market dynamics through the supply contract in a two-echelon supply chain context: a single-supplier and single-retailer system. The uncertainty of the retailer's profits, generated from a supply contract, mainly comes from the stochastic market demand shift, which evolves as a GBM process. In our proposed model, a retailer has rights to sign a supply contract with the supplier at any time to receive goods at a predetermined constant cost rate via the contract. The retailer is incurred with the contracting fees when it decides to reach the supply agreement. Once the market demands are reversed to a downtrend, the retailer can freely determine the time when to stop the contract by paying the cancellation fees to the supplier. After the cancellation, the retailer is still waiting for the next opportunity to grasp the profits generated from the next round of the contracting activity. In short, the retailer owns an entry option when waiting for the investment opportunity, while the retailer exercises the entry option to own a supply contract, it also owns the exit option to quit the supply contract at any time when the market demands are reversed downwards.

To construct the value dynamics of the supply contract in the context mentioned above, this paper extends the combined entry/exit option model proposed by Dixit (1989) to accommodate the stochastic contractual value process, driven by the demand uncertainty. Based on our proposed model, a system of non-linear equations is derived to solve for the option and threshold values. Unfortunately, no analytical solutions are obtained in this system of simultaneous equations. We therefore extend the Levenberg-Marquardt (LM) algorithm to serve as the minimization procedure for the squared estimation errors. After several iterations, an approximated solutions could be found when the estimation errors are less than a very small positive real number, which is arbitrarily chosen in this algorithm.

Based on the numerical results, we conclude that both the volatility of the market demands and the price elasticity of the market demands are positively correlated with the entry/exit option values and the entry/exit thresholds, while the discount rate negatively correlated with them. However, both the option values and the threshold values are more sensitive to the price elasticity of the market demands than to the demand volatility. That is, when the market demands are more volatile, the retailer should wait for the investment opportunity longer because of the greater entry threshold. Based on the entry threshold derived in this paper, the retailer should closely monitor the market conditions and then respond quickly to the stochastic value process to see whether it reaches the level of the entry threshold, at which the retailer should sign the supply contract immediately to activate the goods-delivery from the supplier. Conversely, in a more volatile market condition, the higher exit threshold is also obtained in our model to inform that the retailer will not wait long to exit the contract. Similarly, the price elasticity of the market demands is shown to have the same conclusions as the demand volatility does. However, the option and the threshold values are more sensitive to the price elasticity than to the demand volatility. It implies that the retailer should pay more attention to the price elasticity of demand than to the demand volatility when it makes any decision for the contracting activity. Therefore, the conclusions drawn upon in this model could be employed to assist the retailer in the contracting decisions: the hedging of the demand risks and maximization of the profits.

The entry option discussed in this paper has its implications: the investment opportunity which allows the retailer to wait for the time when the maximum contract value appears. Moreover, if the market demands are reversed downwards, the retailer could save the losses from its waiting or from its abandonment of the supply contract. All the retailer's contracting decisions should be based on the entry/exit thresholds derived in our model. For those readers who interest in the topics of the entry/exit strategies, it is recommended that the assumptions made in this model concerning a single-supplier single-retailer system in the two-echelon supply chain context could be released. A more generalized model could be proposed to explore the conditions on the multi-supplier multi-retailer system in the multi-echelon context. The theory of the option games could be integrated into our model for further studies.

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