THE PROPORTION OF POPULATION METHOD AND OTHER COST ALLOCATION METHODS FOR THE JOINT WATER SUPPLY

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This article uses the joint water supply case to compare the cost allocation methods based on separable costs and non-separable costs: the egalitarian non-separable (ENSC) method, the separable costs remaining benefits (SCRB) method, the minimum costs remaining savings method (MCRS), non-separable cost gap method (NSCG), and a new method, which is called non-separable proportion of population (NSPP). We found the SCRB, MCRS, and NSCG method coincide in the convex cost game case. The ENSC and NSPP method also get the close result. But ENSC method's result is naïve. While the detail cost information is not available, the new method NSPP is a good choice because the method does not need the detail cost information, which is the cost game like MCRS and NSCG need.

1. Introduction

To decrease the water supply cost, the joint water supply are usually set because of economies of scale [1]. In the real world, there may not have a regional authority that could help to build this kind of efficient system, if they do not have fair cost allocation. If the individual do not pay a fair cost, they may decide to build their own water supply plants, and a big loss in efficiency will take place. This shows cost allocation is very important.

The purpose of this article is using the joint water supply case to compare the cost allocation methods based on separable costs and non-separable costs: the egalitarian non-separable (ENSC) method, the separable costs remaining benefits (SCRB) method, the minimum costs remaining savings method, non-separable cost gap method (NSCG), and a new method, which is called non-separable proportion of population (NSPP).

This study is organized as follows. First, we introduce this paper. Second, the selected literature review is shown. The third section explains the theory and model. The next section outlines an empirical example and discussion. The final section concludes this paper.

2 Selected Literature Review

There are some widely used methods from cooperative game theory for the cost allocation problem. They are the Shapley value [2-3], the nucleolus [4-5], variants of the nucleolus [6, 1], and the core and variants of the core [6-8].

The cooperative game is also used as the theoretical basis for the allocation methods based on separable and non-separable costs [9]. In the cooperative game, the separable costs are defined as the difference in cost between the player who joins the coalition and who does not join the coalition [10]. If every player has been assigned its own separable cost, then those remaining costs, which are not separable costs, are called the "non-separable costs".

Some cost allocation methods are based on separable and non-separable costs. They are the egalitarian non-separable cost (ENSC) method, the separable costs remaining benefits (SCRB) method, the minimum costs remaining savings method (MCRS), and the non-separable cost gap (NSCG) method.

Driessen and Tijs [9] compared these four different cost allocation methods based on the joint costs of water resources projects. They argue that:

> *All these methods, except the ENSC method, can be described with the aid of lower and upper bounds for the core of the involved cost game. For convex cost game, these three methods use the same bounds for the core and hence coincide, but their cost allocation does not necessarily belong to the core.*

The separable costs remaining benefits (SCRB) method is used in multipurpose water reservoir projects in the U.S. [11]. The method was widely used in multipurpose water development projects in the 1980s. The disadvantage of the SCRB method is it only analyzes coalitions of size 1, $N – 1$, and N. All other information is ignored. The bounds, upper and lower, for the core are made by simple formulas.

The minimum costs remaining savings method (MCRS) is proposed by Heaney and Dickinson [7]. It is a generalization of SCRB. The bounds for the core are obtained by solving several linear programs. Therefore, in the MCRS method, those bounds are as sharp as possible [9]. The egalitarian non-separable cost (ENSC) method is a naive method. This is because ENSC simply equally assigns the non-separable cost, which should be "proportioned" equally. The non-separable cost gap method (NSCG) is derived by the τ value, which is introduced by Tijs [12]. The method gets the bounds of the core by simple formulas and argues the allocation of the non-separable cost is not only based on the remaining alternate costs of the one-person coalitions, but also needs to be based on the remaining alternate costs of other coalitions.

Heaney and Dickinson [7] argue "... if the problem of regionalization of sewage treatment facilities among *N* cities is being examined, the optimal solution may call for one large plant. In order to realize these savings, the *N* communities must somehow apportion the cost of this regional facility among themselves in a fair manner" [7, p. 476]. That tells us the importance of cost allocation. In this paper, they also compare several criteria of cost allocation: (1) the Shapley value; (2) the nucleolus; (3) the separable costs, remaining benefits (SCRB); and (4) the minimum cost, remaining savings (MCRS) solution. The Shapley value may fall outside the core (which is a very undesirable property) when the game is non-convex. A disadvantage of the nucleolus is that solving $N – 1$ linear programs to get the nucleolus value is rather tedious. For the SCRB method, only coalitions of size 1, $N – 1$, and N are analyzed (other information is ignored).In contrast, all other intermediate coalition information is used in the MCRS method [7]. Therefore, Heaney and Dickinson [7] argue "The MCRS method is preferable to the SCRB method because it uses bounds which are feasible" [7, p. 481].

When economies of scale are present, cooperative game theory is a good method to handle the cost allocations in regional water quality modeling systems. However, the cooperative game theory has a disadvantage: in the real world, there is often not enough information on costs to apply the theory, especially when the region is big and the players are many [6, 13]. Another disadvantage is it is difficult to choose the cost allocation method because no one method is universally preferred [13, 6].

3. Theory and Model

3.1 Theory of N-Person Cooperative Game

Cooperative game theory can be systematically applied to make cooperative decisions [1]. Therefore, fairness must exist between the project members. If one game is seeking the minimum cost, we call this game a cost game. Heaney and Dickinson [7, p. 477] wrote:

> *An N-person game (N, c) in characteristic function form consists of a set N = 1, 2, ..., n of players along with the characteristic function c, which assigns the real number c(S) to each nonempty subset S of players. Cost games are sub-additive, i.e.*

$$c(s) + c(T) \geq c(S \cup T) \text{ for } S \cap T = \varphi \; S, T \subset N. \tag{1}$$

Where φ is the empty set, S and T are any two subsets of N.

There are three general axioms in a cost game to set the fair solution [7, p. 478]. In the first, the costs assigned to the *i*th group, x_i, must be no more than their costs when they act independently, that is,

$$x_i \leq c(i) \qquad \forall i \in N. \tag{2}$$

In the second, the total cost *c*(*N*) must be apportioned among the *n* groups, that is,

$$\sum_{i \in N} x_i = c(N) \tag{3}$$

If the above two equations are satisfied, we call these solution "imputations." In the third, the criterion is extended from the first equation. That means the cost to each member must less or equal to the costs they would receive in any coalition *S* contained in *N*, that is,

$$\sum_{i \in s} x_i \leq c(s) \quad \forall s \in N \tag{4}$$

All solutions satisfied the above three equations. It makes the core of the game. Heaney and Dickinson [7, p. 478] argue:

For sub-additive games the set of imputations is nonempty, but the core may be empty. A cost game has a convex core if

$$c(s) + c(T) \geq c(S \cup T) + c(S \cap T) \text{ for } S \cap T = \varphi \quad S, T \subset N \tag{5}$$

In general, the more attractive (lower cost) the game is, the greater the chance that the core is convex. Conversely, the less attractive the game is, the greater the chance that the core is empty.

3.2 Cost Allocation Methods Based on Separable and Non-separable Costs

For the cost allocation methods based on separable and non-separable costs, we use the ENSC, SCRS, NSCG, MCRS methods, and the NSPP method, which is the method proposed by this paper. In this section we will define what is separable and non-separable cost applied in cost allocation. Then, the above five methods will be explained.

In the cooperative game, the separable costs are defined as the difference in cost between the player who joins the coalition and who does not join the coalition [10]. It is the same as marginal cost and we can define the separable cost of player in a cost game (*N; c*) by

$$SC_i(c) := c(N) - c(N - \{i\}) \tag{6}$$

If every player has been assigned its own separable cost, then those remaining costs, which are not separable costs, are called the "non-separable costs". It can be defined as

$$NSC(c) := c(N) - \sum_{j=1}^{n} SC_j(c) \tag{7}$$

To decide the allocation of the non-separable cost among the plays, a real number $\beta_i(c)$, *i*=1,2,3,…,*n*, can be chosen by the different cost allocation methods.
The egalitarian non-separable cost (ENSC) method is to choose $\beta_i(c)$=1 for all $i \in N$.
This means ENSC simply equally assigns the non-separable cost, which should be "proportioned" equally. Thus this cost method for the cost allocation to player *i is* given by

$$ENSC_i(c) = SC_i(c) + n^{-1} NSC(c) \tag{8}$$

The separable costs remaining benefits (SCRB) method is used in multipurpose water reservoir projects in the U.S. (Interagency Committee on Water Resources, 1958) and it was widely used in multipurpose water development projects in the 1980s. The method chooses $\beta_i(c) = \min(b_i(c), c(\{i\}) - SC_i(c))$ for all $i \in N$, where $b_i(c)$ means the benefit to play *i* in the game (*N; c*) by acting independently.
$c(\{i\}) - SC_i(c)$ is the difference between each player's willingness to pay and the separable cost already

allocated. Since the benefits, $b_i(c)$, usually exceed the alternate cost, $c(\{i\}) - SC_i(c)$, the SCRB method of the cost allocation to player i is usually defined by

$$SCRB_i(c) = SC_i(c) + [c(\{i\}) - SC_i(c)]\left[\sum_{j=1}^{n} c(\{j\}) - SCj(c)\right]^{-1} NSC(c) \tag{9}$$

The SCRB method only analyzes coalitions of size 1, $N - 1$, and N. All other information is ignored. Driessen and Tijs [9] proposed the remaining alternate costs of other coalition of the non-separable cost need to be included. Therefore, they introduce the cost gap function g^c. For each coalition S, the cost gap of S in the cost game $(N; c)$ is defined as

$$g^c(S) = c(S) - \sum_{j \in s} SC_j(c) \quad \text{if } S \neq \phi \quad \text{and} \quad g^c(\phi) := 0 \tag{10}$$

Since the cost gap function is nonnegative, we assume $g^c(S) \geq 0 \ \ \forall S \subset N$.

Let T be a coalition that player i belongs to. In any cost allocation, player i will reject paying more than $SC_i(c) + g^c(T)$ .This is because player i will threaten to set up the coalition T where all the members except player i will be charged only their separable costs, while player i will be charged the remaining cost which is $C(T) - \sum_{j \in T-(i)} SC_j(c)$ equaling $SC_i(c) + g^c(T)$.The above argument applies to any coalition T including player i. Therefore player i will not pay more than the amount $\min_{T;i \in T}[SC_i(c) + g^c(T)]$ which equals $SC_i(c) + \min_{T;i \in T} g^c(T)$. From the above argument, we can define the concession amount of player i in a cost game $(N;c)$ by

$$\lambda_i(c) = \min_{T;i \in T} g^c(T) \tag{11}$$

The above equation shows the concession amount $\lambda_i(c)$ of player i can be seen as maximal contribution to the non-separable cost $NSC(c)$. Therefore, we assume the total of the maximal contribution is not less than the non-separable cost. That is

$\sum_{j=1}^{n} \lambda_j(c) \geq NSC(c)$. The non-separable cost gap (NSCG) method uses the players' proportional concession amount to allocate the non-separable cost. Therefore, if $g^c(S) \geq 0 \ \ \forall S \subset N$ and $\sum_{j=1}^{n} \lambda_j(c) \geq NSC(c)$ are satisfied. The NSCG method of cost allocation for player i is defined as

$$\begin{aligned} NSCG_i(c) &= SC_i(c) \quad & NSC(c) = 0 \\ &= SC_i(c) + \lambda_i(c)\left[\sum_{j=1}^{n} \lambda_j(c)\right]^{-1} NSC(c) \quad & NSC(c) > 0 \end{aligned} \tag{12}$$

The MCRS method, which is proposed by Heaney and Dickinson [7], is a generalization of SCRB. The lower and upper bounds used in the method for the core are as sharp as possible. If games *(N;c)* have a nonempty core, we find the upper and lower bounds on each x_i, cost allocation of player *i*, by

$$\text{Maximize or minimize } x_i \tag{13a}$$

subject to:

$$x_i \le c(\mathrm{i}) \qquad \forall i \in N \tag{13b}$$

$$\sum_{i \in s} x_i \le c(s) \quad \forall\, s \in N \tag{13c}$$

$$\sum_{i \in N} x_i = c(N) \tag{13d}$$

$$x_i \ge 0 \qquad \forall i \in N. \tag{13e}$$

Therefore *MCRS* method of cost allocation for player *i* is defined as

$$MCRS_i(c) = x_i^{\min} + u_i(c)\left[c(N) - \sum_{i \in N} x_i^{\min}\right]$$

where

$$u_i(c) = \left[x_i^{\max} - x_i^{\min}\right]\left[\sum_{i \in N}\left(x_i^{\max} - x_i^{\min}\right)\right]^{-1} \tag{14}$$

For the NSPP method, the non-separable cost is proportioned by the player's share of their population. Thus the NSPP method of the cost allocation to player *i is* defined by

$$NSPP_i(c) = SC_i(c) + Q_i(c)\left[\sum_{i \in N} Q_i(c)\right]^{-1} NSC(c) \tag{15}$$

where Q_i is the quantity per capital. The advantage of this method is that it does not need detail information (information of all the subgroups), which the MCRS and NSCG need, of the cooperative game.

4. *Empirical Example and Discussion*

4.1 data

The data used in the paper is the example given by Young [6, p. 464]. In the example, there are three neighboring municipalities 1,2, and 3. They build a joint water supply facility to supply themselves with municipal water.

To compare the above five cost allocation methods, we consider these three municipalities cost game *(N;c)* with

C(1)= 6.5 C(12)= 10.3 C(123)= 10.6

C(2)= 4.2 C(13)= 8.0

C(3)= 1.5 C(23)= 5.3

The annual rates of water use of supplying water in the three municipalities given in the table 1 can be used to compute the proportion of the population.

Table 1. Annual Rates of Water Use of Supplying Water in Three Hypothetical Municipalities

Municipality	Use per Capita, m^3
A	140
B	120
C	120

* The above information is from Young et al [6, p. 464], table 1

4.2 Results and Discussion

The results of cost allocation are shown in table 2. We use software GAMS to solve any methods with linear programs. From table 2, we not only can see the cost allocation of any municipality is less than the cost they work individually but also all the cost allocation is in the core (see the equation (2),(3), and (4)). This means the coalition is stable and no one will leave the grand coalition (3 municipalities) to form the subset of the 3 municipalities. Therefore, the coalition is efficient and equal.

The example used in the paper is a convex cost game (see the equation (5)), which has very nice properties in the game theory literature. In table 2, the SCRB, NSCG, and MCRS method coincide. That is because they may use the same bounds in the convex cost game. The ENSC and NSPP method show a close result with the above three methods. But the ENSC is less important, since it simply assigns the non-separable cost.

For the new method NSPP, it may a good choice. Although the NSCG and MCRS methods use the information of all coalitions and get the sharper bounds for the core, we may not get all the information, for example, there are too many players.

TABLE 2 .Comparative Results for these five Cost Allocation Methods

Municipality	1	2	3	Total
$ENSC_i(c)$	6.10	3.40	1.10	10.6
$SCRB_i(c)$	6.02	3.56	1.02	10.6
$NSCG_i(c)$	6.02	3.56	1.02	10.6
$MCRS_i(c)$	6.02	3.56	1.02	10.6
$NSPP_i(c)$	6.18	3.36	1.06	10.6

(Unit: dollars $\times 10^6$)

5.Conclusion

In the joint water supply case, all these five cost allocation methods based on separable costs and non-separable costs solve the efficiency and equity problem. Using the convex cost game, we found the NSPP method proposed in the paper has stable cost allocation in the core. The method is a good choice when the coalition information can not be too detailed. For the future study, we may use the non-convex cost game and many players (greater than four) to test the NSPP method to see whether the method still gives a good result.

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