Self-Organized Model Approach to Aggregate Fluctuations in Production Activity of Firms

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Scheinkman and Woodford along with Bak and Chen proposed a self-organizing model for production activity of firms to illustrate how large fluctuations in aggregate economic activity can be triggered by small independent shocks. This paper develops their model to accommodate more realistic production networks, and examines numerically influences caused by the change of the network structure.

Keywords: Economic Fluctuations; Self-Organized Criticality; Production Network

1. Introduction

The model of Bak, Chen, Scheinkman and Woodford considered the production activity to be a chain reaction of orders by firms.[1–3] It was assumed that the firms connect through a regular network with some amount of inventories. The network defines a flow of order from the top layer to the bottom layer. When a firm receives an order from upstream firms, it produces its own products. And if it suffer from shortage of inventories, it sends an order to downstream firms. Endogeneous orders to the top layer arrive randomly, and will sometimes produce a chain reaction of orders for intermediate goods like an avalanche. This mechanism of self-sustained economic fluctuations is akin to that of self-organizing behavior of sand pile.

Although the model is very attractive, it heavily depends on the regularity of production network and the hierarchy of order flow. It seems to us that these constrains are too restrictive in light of real economy. In fact, it has been pointed out recently that connections between firms have characteristics of a scale-free network.[6,7] The scale-free network has a neither regular nor layered structure at all. The purpose of the present paper is therefore to generalize the original model to study more realistic patterns of connection among firms.

2. Simulations on Various Networks

We measure the size of such a chain reaction by the total number of activated firms. The size of an avalanche depends on the level of inventory. If firms have few inventories, each order from upstream firms will give rise to orders to downstream firms, so that the avalanche size will become large and the inventory level of the system increases. On the other hand, if firms have sufficient inventories, it readily stops the order flow. The avalanche size tends to be small and the the inventory level decreases. This back-and-forth mechanism lead a self-organized economy with a critical level of inventories irrespective of initial conditions.

2.1. *Regular Network*

We first revisit the original results on a regular network as shown in Fig. 1 with an example of order flow. Fig. 2 is a result of a simulation for a 1600×1600 lattice (256×10^4 firms). It shows the cumulative distribution of the size of avalanches with both axes in the logarithmic scales. This distribution was derived out of 10^5 samples. The avalanche size follows a power-law distribution with exponent 1/3. This result was obtained analytically.[4] The rapid attenuation arises from finiteness of the network.

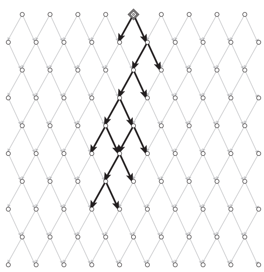

Fig. 1. The diagram of the order flow on the regular network system

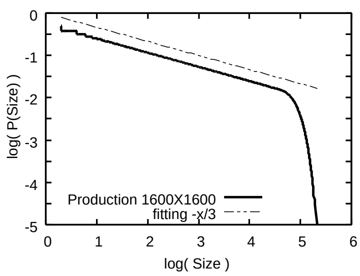

Fig. 2. The distribution of avalanche size on the regular network (logscale)

2.2. *Random Network with a Layer Structure*

It is unnatural to assume firms in the economy connect in such a regular way. Additionally, the power-law exponent of 1/3 is too small. The power-law exponent is close to or more than 1 in a wide variety of economic and social systems (e.g. distribution of population of cities, production scale of firms).

Here we generalize the model for firms so as to connect through a random network, and examine to what extent change of the network structure influence on the production fluctuations. In the original model, firms are connected to two adjacent firms on the upper and lower layers. On the other hand, in this model, firms are connected to two firms chosen randomly in both layers; Fig. 3 illustrates an order flow.

Fig. 4 is a result of a simulation with the same condition as the previous simulation. The distribution of the avalanche size is well fitted by the straight line corresponding to the power-law with an exponent1/2. We thus see that the self-organized critical state qualitatively depends on the network adopted.

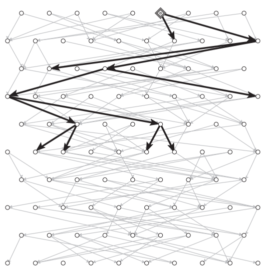

Fig. 3. The diagram of the order flow on the random network system

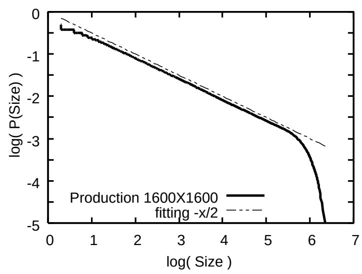

Fig. 4. The distribution of avalanche size on the random network (logscale)

2.3. *Completely Random Network*

In the previous models, orders flow layer by layer. If the order flow is directed randomly, that is, the layer structure is destroyed, the avalanche size does not show a power-law distribution and large avalanches are appreciably depressed.

2.4. *Scale-Free Network*

To make the model more realistic, we incorporate scale-free nature into the production network. The scale-free network has the character that the degree distribution for nodes obeys the power law. The network used here was generated by preferential attachment of nodes;[5] the degree distribution is controllable. Fig. 5 is an example of order flow on such a scale-free network.

Fig. 6 is the cumulative distribution of the avalanche size on the scale-free network obtained by a simulation with 256×10^4 firms and 10^5 samples.

The result shows a power-law distribution for the avalanche size. We note that its exponent (≈ 2) is close to that ($= 2$)of the degree distribution of the given network. We carried the same simulations with varied exponents for the degree distribution of the scale-free network. Then we confirmed that the power-law exponent of the distribution of the avalanche size changed in accordance with that of the degree distribution.

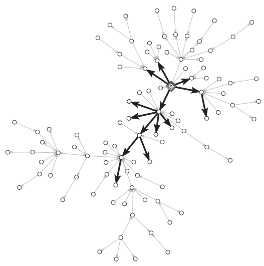

Fig. 5. The diagram of the order flow on the scale-free network system

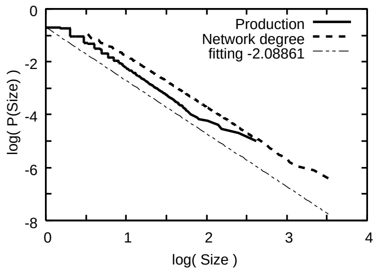

Fig. 6. The distribution of avalanche size on the scale-free network (logscale)

3. Conclusion

We have generalized the model due to Bak *et al.* to take into account the scale-free nature of the production network recently observed. We then carried out simulations for varied structures of the network and demonstrated numerically how the distribution of the economic fluctuations were influenced by the change of the network structure.

References

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