The Impact of Asymmetric Belief and Bounded Rationality on Market Price Volatility *

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Extended Abstract

Financial market volatility is essential the driving force of financial markets. The understanding and management of the volatility are the focus of finance literature, in particular, the mechanism of market volatility. From the early representative agent and rational expectation theory to the current heterogeneous beliefs and bounded rationality framework, we have witnessed an extensive development along the line and we refer to some of the recent survey papers by Hommes (2006), LeBaron (2006) and Chen (2007).

Most of the heterogeneous agent models (HAMs) have been developed successfully to explain some complex phenomena in financial markets, such as market booming and crashing, and the stylized facts, such as fat tail, long memory, volatility clustering and excess volatility. The beliefs can be either linear or nonlinear but symmetrical about price change. Implicitly this assumes that investors behave in the same way when market is either booming or crashing. However, this assumption is not realistic. Very often, investors behave differently when the market condition changes and different market volatility patterns are observed. In this paper, within the HAM framework, we model the asymmetric behaviour of agents when market is in different condition and examine the impact of asymetric belief on the market price dynamics, in particular the volatility.

In this paper, we consider a market with one risky asset (such as stock, index or managed fund) with (ex dividend) price P_t and one risk free asset with fixed gross rate $R \geq 1$. The market consists of heterogeneous belief investors. Without loss of generality, we consider there are two types of investors, fundamentalists and chartists (f and c). Similar to Chiarella, He & Hommes (2006), we assume that the risky asset

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price in each period is set via a market maker mechanism upon the aggregate excess demand $D_t = n_{f,t}D_t^f + n_{c,t}D_t^c$. However, to avoid possible negative prices, we assume the adjustment mechanism the market maker is in relative price change rather than the absolute amount, that is

$$\frac{P_{t+1} - P_t}{P_t} = \mathcal{S}(D_t) = \mathcal{S}\left(\sum_{q\in\{f,c\}} n_{q,t}D_t^q\right), \tag{1}$$

where $\mathcal{S}(\cdot)$ is an S-shaped function, D_t^q is the excess demand of type q traders at time t and $n_{q,t}$ is the market fraction of type q traders at time t satisfying $n_{f,t} + n_{c,t} = 1$.

In terms of the heterogeneous beliefs, we assume that fundamentalists know the fundamental information of the risky asset, such as the fundamental price. They believe that the price may be away from the fundamental value for a while but in long term, the price will revert to its fundamental. So their demand is based on the spread between the actual price P_t and the fundamental price F_t, which can be expressed as $D_t^f = \alpha(F_t - P_t)$ where $\alpha > 0$.

In contrast, the chartists do not have the fundamental information. They prefer cheaper thumb strategies by believing that the charting signals from the past price can be used to forecast the future price movements. In this paper, the price trend for the chartists is assumed to be expressed by the weighted historical prices, that is $\widehat{P}_t = (1-\omega)P_{t-1} + \omega P_{t-2}$, where $\omega \in [0,1]$ represents the coefficient of the retracement strategy. Thus a trading signal to the chartists is defined as $\delta_t = P_t - \widehat{P}_t$. When $\delta_t > 0$, the chartists believe that the price is in an increasing trend and hence they want to hold a long position; otherwise, they will take a short position. Specifically, the demand of the chartists can be written as $D_t^c = g(\delta_t)$. Here g is usually assumed to be an S-shaped function, for example $g(x) = \mathbb{u}\tanh(\mathbb{v}x)(\mathbb{u} > 0,\ \mathbb{v} > 0)$. Note that under the assumption, $g(\cdot)$ is symmetric and $\lim_{x\to\pm\infty} |g(x)| = \mathbb{u} < \infty$, which means that the actions of the chartists to both the positive and negative trading signals are completely equally reverse operations and the chartists are cautious when the price difference δ_t is large (either positive or negative), but not very cautious. Given that the chartists are less informed, they become less confident and cautious when there are big price difference in general. To characterize such behaviour, we assume that the function g has the following general properties: $\exists x_1^* < 0,\ x_2^* > 0$ such that

$$g(0) = 0,\ xg(x) > 0 \text{ for } x \neq 0, \tag{2a}$$

$$g'(x) > 0 \text{ for } x_1^* < x < x_2^*; \quad g'(x) < 0 \text{ for } x < x_1^* \text{ or } x > x_2^*, \tag{2b}$$

$$\lim_{x\to x_1^*} g(x) = g_l < +\infty, \quad \lim_{x\to x_2^*} g(x) = g_u > -\infty. \tag{2c}$$

In particular, in this paper, we take $g(x) = \frac{ax}{1+bx+c^2x^2}$, where $a > 0,\ c > 0,\ b \in (-2c, 2c)$, which satisfies the characteristics in (2). Here b measures the asymmetry of the chartists' response to changes of x, especially when $b < 0 (> 0)$, the chartists believe a bullish (bearish) market. The parameter $|x^*_{1,2}| = 1/c$ represents the confident level and if the absolute change of the price is beyond $1/c$, then the chartists become cautious by reducing their positions.

Similar to Brock & Hommes (1997), we consider traders can change their strategies based on a weighted average of net realized profit

$$U_{q,t} = (P_t + y_t - RP_{t-1})D^q_{t-1} - C_q + \eta U_{q,t-1}, \quad q \in \{f, c\},$$

where $C_q \geq 0$ is the cost type q investors should pay for their investment, $\{y_t\}$ is the dividend process of risky asset and the parameter $\eta \in [0, 1)$ represents the memory of the cumulated fitness function. Then the updated fraction $n_{q,t}$ is given by the discrete choice probability $n_{q,t} = e^{\beta U_{q,t}}/N_t$, where $N_t = \sum_{q\in Q} e^{\beta U_{q,t}}$ and the parameter $\beta(\geq 0)$ is the sensitivity of performance measuring on how fast the fractions of the different type agents in the market switch each other. Summarizing the above analysis and letting $C = C_f - C_c$ and $U_t = U_{f,t} - U_{c,t}$, then we obtain the following model

$$\begin{cases} P_{t+1} = P_t\left[1 + \mathcal{S}\left(\dfrac{e^{\beta U_t}}{e^{\beta U_t}+1}\alpha(F_t - P_t) + \dfrac{1}{e^{\beta U_t}+1}g(P_t - (1-\omega)P_{t-1} - \omega P_{t-2})\right)\right] \\ U_{t+1} = (P_{t+1} + y_{t+1} - RP_t)\Big[\alpha(F_t - P_t) - g\big(P_t - (1-\omega)P_{t-1} - \omega P_{t-2}\big)\Big] - C + \eta U_t. \end{cases} \quad (3)$$

In this paper, we first use stability and bifurcation theory of nonlinear dynamical systems to analyze (3) with constant fundamental price and dividend and to examine the stability of the fundamental steady state. Over the last two decades, economists have become familiar with period doubling and Hopf bifurcations. These are examples of codimension one bifurcations. Economic applications of bifurcations of codimension two or higher are rare due to the fact that they are more difficult to handle, except Benhabib, Schmitt-Grohé & Uribe (2001) and Gaunersdorfer, Hommes & Wagener (2006). In this paper, we find some important codimension two bifurcations of the model, such as generalized flip bifurcation, Chenciner bifurcation and $1 : q$-resonance ($q = 2, 3, 4$). A codimension two bifurcation is a generic phenomenon when two parameters are varied simultaneously. In our paper, we emphasize particularly on the effect of the extrapolation rates of the fundamentalists and chartists (α and a) and asymmetric belief and confidential level of the chartists (b and c).

By analyzing the linear and nonlinear parts of system (3), we explicitly give a sufficient condition of generalized flip bifurcation, Chenciner bifurcation and $1 : q$-resonance. From those explicit criteria, we find that the extrapolation rates of the fundamentalists and chartists determine the stability of the fundamental steady state. However, the asymmetric belief (b) and confidential level ($1/c$) of the chartists play a very important role on how the stability of the fundamental equilibrium is broken. For example, when the chartists are symmetric about bullish and bearish market conditions ($b = 0$) or they are so confident ($1/c$ sufficiently large), then there is no Chenciner bifurcation. In other words, a stable fundamental steady state and a stable invariant circle cannot coexist.

However, when the chartists have a bullish belief ($b < 0$) or bearish belief ($b > 0$), price fluctuation may increase and the market may move to the stable fundamental value or a stable invariant cycle depending on the initial value.

One of the important findings of codimension two bifurcations for our evolutionary learning model is that, close to a bifurcation value, there is an open region in the parameter space where a stable fundamental steady state and another stable attractor, such as an invariant circle or a period-two cycle, coexist. In this region, the price dynamics depend on the initial states of the system. Such a region is called a "volatility clustering region" according to Gaunersdorfer, Hommes & Wagener (2006). In addition, some codimension two bifurcations like $1:q$-resonance lead to complex phenomena and even chaos, which have potential to explain some stylized facts in financial markets.

Through stochastic simulation, we show that the statistical results of the corresponding stochastic model reflect closely the stylized features observed in financial markets, including volatility clustering, high kurtosis, and the long-range dependence of asset returns.

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