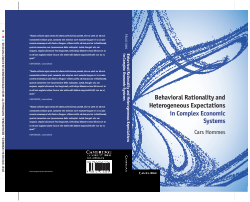
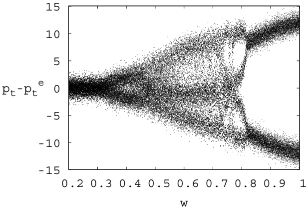
Complex Economic Systems in Macro & Finance
Lecture I: Application Cobweb Model

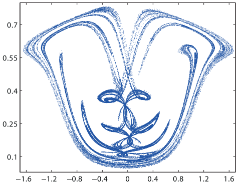
Cars Hommes

CeNDEF, University of Amsterdam

CEF 2015 Workshop
Taipei, Taiwan, June 2015







Outline

1. Nonlinear Cobweb model with Homogeneous Beliefs (Chapter 4)
   - Naive expectations
   - Rational expectations
   - Adaptive expectations

2. Cobweb Model with Heterogeneous Beliefs (Chapter 5)
   - Rational versus naive
   - Evolutionary Selection and Reinforcement Learning

Cobweb ('hog cycle') Model

- market for non-storable consumption good (e.g. corn, hogs)
- **production lag**; producers form price expectations one period ahead
- partial equilibrium; market clearing prices

p_t^e: producers' price expectation for period t
p_t: realized market equilibrium price p_t

Cobweb (‘hog cycle’) Model (continued)

$$D(p_t) = a - dp_t(+\epsilon_t) \qquad a \in R, \quad d \geq 0 \qquad \text{demand} \qquad (1)$$

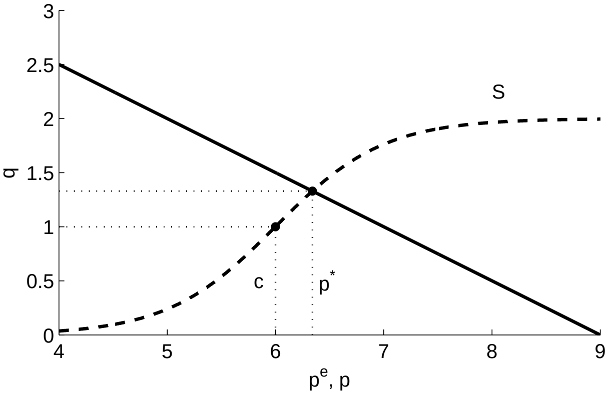
$$S_\lambda(p_t^e) = \tanh(\lambda(p_t^e - 6)) + 1, \qquad \lambda > 0, \qquad \text{supply} \qquad (2)$$

$$D(p_t) = S_\lambda(p_t^e) \qquad \text{market clearing} \qquad (3)$$

$$p_t^e = H(p_{t-1}, ..., p_{t-L}), \qquad \text{expectations} \qquad (4)$$

Price dynamics: $p_t = D^{-1}S_\lambda(H(p_{t-1}, ..., p_{t-L}))$
Expectations Feedback System:
dynamical behavior depends upon expectations hypothesis;
supply driven, **negative feedback**

Demand and (nonlinear) Supply in Cobweb Model

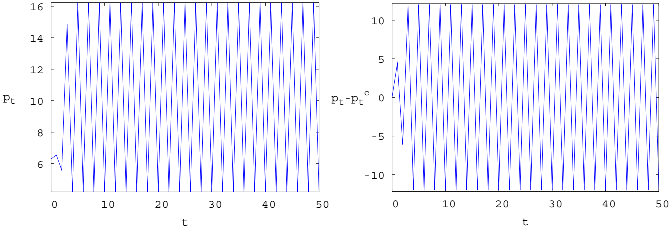


Expectations

- **naive** expectations: $p_t^e = p_{t-1}$
- **adaptive** expectations; $p_t^e = wp_{t-1} + (1-w)p_{t-1}^e$
- **backward looking average** expectations $p_t^e = w_1 p_{t-1} + wp_{t-2}$
- **rational** expectations: $p_t^e = E_t[p_t] = p^*$

Naive Expectations Benchmark ($p_t^e = p_{t-1}$)

unstable steady state iff $S'(p^*)/D'(p*) < -1$



Regular **period 2 price cycle** with **systematic forecasting errors**

Agents will **learn** from their mistakes and **adapt forecasting behavior**

Rational Expectations (Muth, 1961)

Expectations are **model consistent**
all agents are rational and **compute** expectations from market equilibrium equations

$$p_t^e = E_t[p_t] \qquad \text{or} \qquad p_t^e = p_t \qquad \text{or} \qquad p_t^e = p^*$$

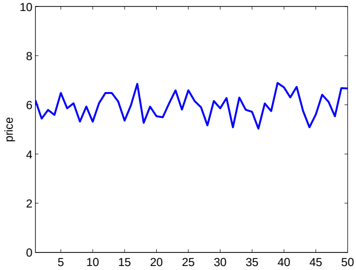
implied **self-fulfilling** RE price dynamics

$$p_t = p^* + \delta_t$$

perfect foresight, **no systematic forecasting errors**

Important Note: this is impossible in complex, heterogeneous world

Rational Expectations Benchmark ($p^* = 5.93$)



Problem: need perfect knowledge of "law of motion" and high computing abilities

Adaptive Expectations ("error learning")

Nerlove 1958

$$\begin{aligned} p_t^e &= (1-w)p_{t-1}^e + wp_{t-1} \\ &= p_{t-1}^e + w(p_{t-1} - p_{t-1}^e) \\ &= wp_{t-1} + (1-w)wp_{t-2} + \cdots (1-w)^{j-1}wp_{t-j} + \cdots \end{aligned}$$

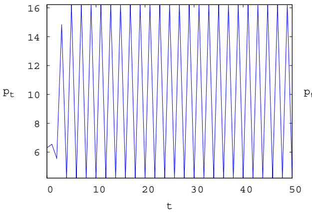
weighted average of past prices

1-D (expected) price dynamics: $p_t^e = wD^{-1}S(p_{t-1}^e) + (1-w)p_{t-1}^e$

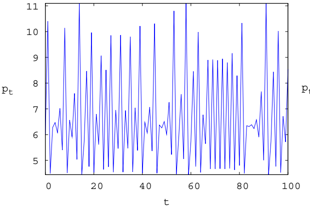
stable steady state if $-\frac{2}{w} + 1 < \frac{S'(p^*)}{D'(p^*)} (< 0)$

- more **stabilizing** in **linear** models, but
- possibly low amplitude **chaos** in nonlinear models

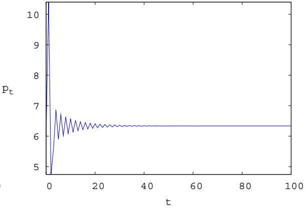
Adaptive Expectations may lead to Chaos



$w = 1$



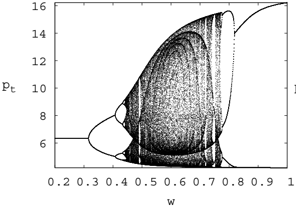
$w = 0.5$



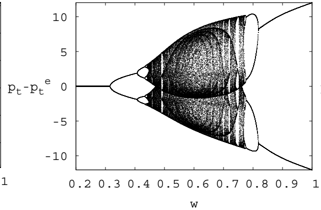
$w = 0.3$

weighted average of nonlinear monotonic D/S curves leads to non-monotonic chaotic map

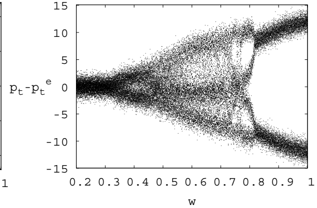
Adaptive Expectations lead to Chaotic Forecast Errors



chaotic prices



chaotic errors



noisy errors

In a **nonlinear** world, adaptive expectations may lead to (small) **chaotic** forecasting errors

Non-monotonic chaotic map for monotonic D and S

C.H. Hommes / Journal of Economic Behavior and Organization 24 (1994) 315–335 327

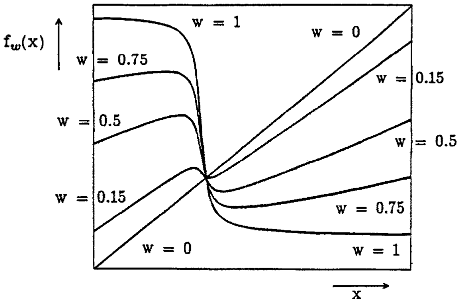


Fig. 4. Graphs of the map f_w for different values of the expectations weight factor w, with $a=0.8$, $b=0.25$, and $\lambda=4$. For w close to 0 f_w has a globally stable equilibrium. For w close to 1 f_w has a stable period 2 cycle. For w close to 0.5 the map f_w is chaotic.

Adaptive Expectations in a Nonlinear World

- adaptive expectations is **stabilizing** in the sense that it reduces the amplitude of price fluctuations and forecast errors
- small amplitude **chaotic** price fluctuations may arise around the unstable steady state
- forecast **errors** may be **chaotic**, highly irregular, with little systematic structure
- in a **nonlinear** world, adaptive expectations may be a **behaviorally rational** strategy for **boundedly rational** agents

Ken Arrow on Heterogeneous Expectations

"*One of the things that microeconomics teaches you is that individuals are not alike. There is* **heterogeneity**, *and probably* **the most important heterogeneity here is heterogeneity of expectations**. *If we didn't have heterogeneity, there would be no trade. But developing an analytic model with heterogeneous agents is difficult.*"

(Ken Arrow, In: D. Colander, R.P.F. Holt and J. Barkley Rosser (eds.), The Changing Face of Economics. Conversations with Cutting Edge Economists. The University of Michigan Press, Ann Arbor, 2004, p. 301.)

Cobweb Model with Homogeneous Expectations

- Demand: $D(p_t) = a - dp_t(+\epsilon_t), \quad a \in R,\ d \geq 0.$
- Supply: $S_\lambda(p_t^e) = sp_t^e, \quad s > 0.$
- Market clearing: $D(p_t) = S_\lambda(p_t^e).$
- Expectations: $p_t^e = H(p_{t-1}, ..., p_{t-L}).$

- **Price dynamics**: $p_t = D^{-1}S_\lambda(H(p_{t-1}, ..., p_{t-L})).$
- Note: linear supply curve derived from profit maximization with **quadratic cost function** $c(q) = q^2/(2s).$

Cobweb Model with Heterogeneous Beliefs I.

- Market clearing:

$$a - dp_t = n_{1t} s p^e_{1t} + n_{2t} s p^e_{2t} \quad (+\epsilon_t),$$

where n_{1t} and $n_{2t} = 1 - n_{1t}$ are **fractions** of the two types.

- Forecasting rules:
  1. **rational**: $p^e_{1t} = p_t$,
  2. **naive**: $p^e_{2t} = p_{t-1}$.

- Information gathering **costs**: rational - $C > 0$; naive - free.

Cobweb Model with Heterogeneous Beliefs II.

- **Market clearing** becomes:

$$a - dp_t = n_{1t} s p_t + n_{2t} s p_{t-1} \qquad (+\epsilon_t)$$

- **Price dynamics**:

$$p_t = \frac{a - n_{2t} s p_{t-1}}{d + n_{1t} s}.$$

- How do **fractions** change over time?

Evolutionary or Reinforcement Learning

- Agents can choose between different types of forecasting rules.
- Sophisticated rules may come at information gathering costs $C > 0$ (Simon, 1957), simple rules are freely available.
- Agents evaluate the net past performance of all rules, and **tend to follow rules that have performed better in the recent past**.
- Evolutionary fitness measure $\equiv$ past realized net profits.

Discrete Choice Model

- **Fitness Measure**: random utility

$$\tilde{U}_{ht} = U_{ht} + \epsilon_{iht},$$

U_{ht}: deterministic part of fitness measure,
ϵ_{iht}: idiosyncratic noise, IID, extreme value distr.

Fractions of Belief Types

- **Discrete choice** or **multi-nomial logit** model:

$$n_{ht} = e^{\beta U_{h,t-1}} \; / \; Z_{t-1},$$

  where $Z_{t-1} = \sum e^{\beta U_{h,t-1}}$ is a normalization factor.

- β is the **intensity of choice**, inversely related to SD idiosyncratic noise: $\beta \sim 1/\sigma$.

- $\beta = 0$: all types equal weight (random choice).
- $\beta = \infty$: "neoclassical limit", i.e. **all** agents choose **best** predictor.

Fitness Measure

- **Evolutionary Fitness Measure**:
  weighted average of **past realized net profits**

$$U_{ht} = \pi_{ht} + wU_{h,t-1}$$

- π_{ht} net realized profit (minus costs) strategy h.
- w measures **memory strength**
  - $w = 1$: infinite memory; fitness $\equiv$ accumulated profits,
  - $w = 0$: memory one lag; fitness most recently realized net profit.

Fitness Measure Profits

- **Profits** of type *h*:

$$\pi_{ht} = p_t s p^e_{ht} - \frac{(s p^e_{ht})^2}{2s}.$$

- Profits of **rational** agents:

$$\pi_{1t} = \frac{s}{2} p_t^2 - C$$

- Profits of **naive** agents:

$$\pi_{2t} = \frac{s}{2} p_{t-1}(2p_t - p_{t-1})$$

Profit difference

- **Difference** in profits:

$$\pi_{1t} - \pi_{2t} = \frac{s}{2}(p_t - p_{t-1})^2 - C.$$

- **difference** in fractions:

$$m_{t+1} = n_{1,t+1} - n_{2,t+1} = \mathrm{Tanh}(\tfrac{\beta}{2}[\pi_{1t} - \pi_{2t}])$$

$$= \mathrm{Tanh}(\tfrac{\beta}{2}[\tfrac{s}{2}(p_t - p_{t-1})^2 - C])$$

- When the **costs** for rational expectations **outweigh** the **forecasting errors** of naive expectations, more agents will buy the RE forecast.

2-D dynamic system

- Pricing equation :

$$p_t = \frac{a - n_{2t} s p_{t-1}}{d + n_{1t} s} = \frac{2a - (1 - m_t) s p_{t-1}}{2d + (1 + m_t) s}.$$

- Evolutionary selection

$$m_{t+1} = \mathrm{Tanh}(\frac{\beta}{2}[\frac{s}{2}(p_t - p_{t-1})^2 - C]).$$

- **Note Timing**:
  1. Old fractions determine market prices.
  2. Realized market prices determine new fractions.

2-D dynamic system in deviations

deviation from RE fundamental price:

$$x_t = p_t - p^*$$

$$x_t = \frac{-(1-m_t)sx_{t-1}}{2d + (1+m_t)s}$$

$$m_{t+1} = \mathrm{Tanh}(\frac{\beta}{2}\,[\,\frac{s}{2}(x_t - x_{t-1})^2 - C\,]).$$

Properties of the 2-D Dynamics

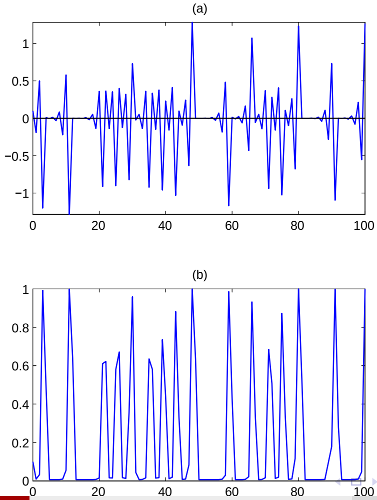
- If all agents are **rational**, then $p_t \equiv p^* = a/(d+s)$.
- If all agents are **naive**, then $p_t = \frac{a - sp_{t-1}}{d}$.
- Unique **steady state** $E = (p^*, m^*)$, $m^* = \mathrm{Tanh}(-\beta C/2)$.
- If $p_{t-1} = p^*$, then $p_t = p^*$ and $m_t = m^*$:
  stable manifold contains vertical line through steady state.
- If $s/d < 1$, then **globally stable** steady state

"Neo-classical" limit case: $\beta = \infty$

- If $s/d > 1$, $C > 0$ and $\beta = +\infty$ then
  - Steady state is **locally unstable saddle point**.
  - Steady state is **globally stable**.
- **Important note:** homoclinic orbits!!

Chaotic Dynamics

Irregular switching between cheap destabilizing free riding and costly sophisticated stabilizing predictor

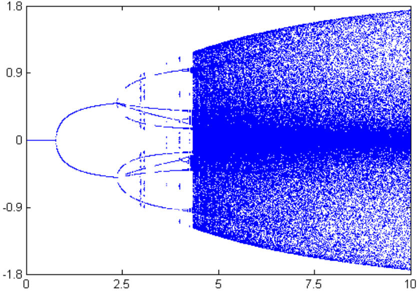


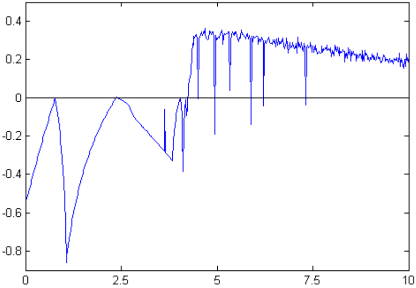
Rational Route to Randomness

If $s/d > 1$ and $C > 0$, then

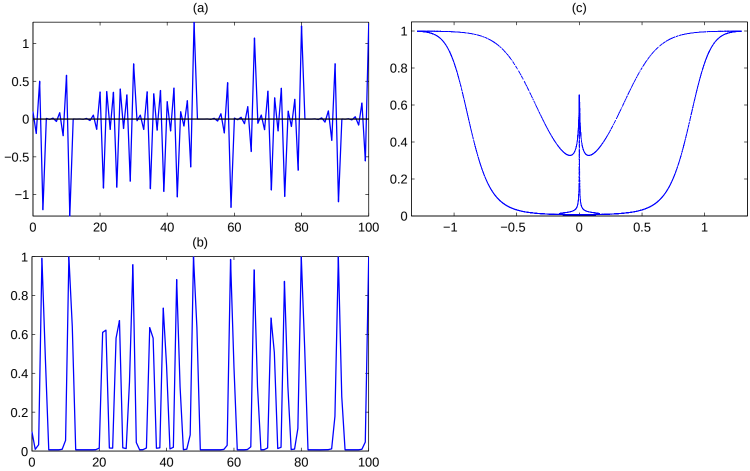
- $0 \leq \beta < \beta_1^*$: stable steady state
- $\beta = \beta_1^*$: **period doubling bifurcation**
- $\beta_1^* \leq \beta < \beta_2^*$: stable 2-cycle
- $\beta = \beta_2^*$: secondary period doubling bifurcations
- $\beta_2^* < \beta^* < \beta_3^*$: two **co-existing** stable 4-cycles
- $\beta > \beta_3^*$: complicated **chaotic dynamics**, strange attractors

Rational versus naive: Rational Route to Randomness

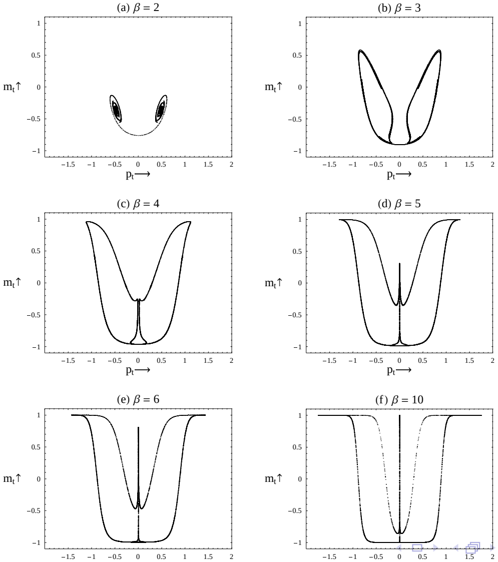




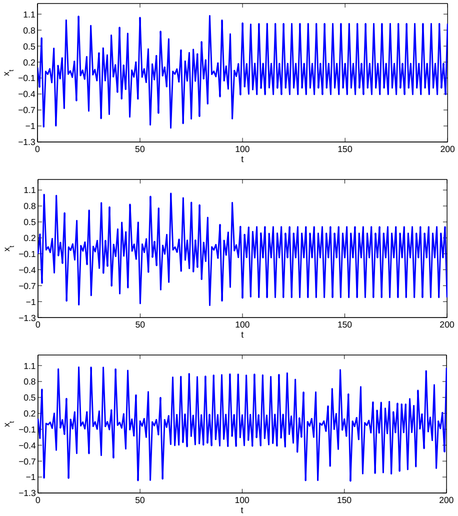
Rational versus naive: time series + attractors



Rational versus naive: unstable manifolds

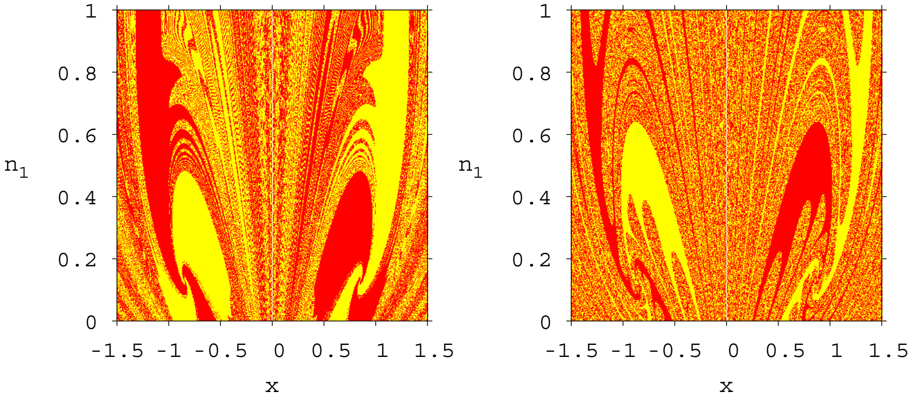


Rational versus naive: two co-existing stable 4-cycles



Basins of Attraction of two coexisting stable 4-cycles

fractal basin boundaries



Summary Nonlinear Cobweb Model

- in **nonlinear** cobweb model with monotonic demand and supply, simple expectation rules may generate **chaos** in prices and errors;
- simple rules in a nonlinear world may be **behaviorally rational**
- **heterogeneous expectations** driven by recent performance may lead to homoclinic bifurcations and chaos
- **simple rules survive evolutionary competition**, especially when more sophisticated rules are costly